Evaluating Aesthetics for User-Sketched Layouts of Clustered Graphs with Known Clustering Information

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Abstract

This paper aims to empirically analyze the aesthetics for user-sketched layouts of clustered graphs with known clustering information. In our experiments, given not only the adjacency list of a clustered graph but also its predefined clustering information, each participant was asked to manually sketch clustered graphs "nicely" from scratch on a tablet system using a stylus. Different from previous works, the main concern in this paper is on which graph drawing aesthetics people favor when sketching their own drawings of clustered graphs with known clustering information. Another concern of this paper is on the aesthetics of clustered graph layouts employed by participants which include not only characteristics and structures of the final graph layouts but also the behavior of user's sketching process (including layout creation and adjustment). By observing all layouts and drawing processes, the drawing strategies which participants applied and the drawing aesthetics are analyzed. Results show that most participants were unsurprisingly able to draw graphs with clear presence of bridge edges and clustering cohesiveness; more importantly, to distinguish clusters within the restricted-size tablet screen during the drawing process, some of the participants were still able to make each cluster with fewer edge crossings, more symmetries, and

☆A preliminary work was presented at 8th International Symposium on Visual Information Communication and Interaction (VINCI 2015), Aug. 24-26, 2015, Tokyo, Japan.

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more alignment of grid in a smaller drawing area where the cluster spreads. Our results support that to alleviate user's complex drawing tasks, aside from the grid-based editing function suggested by the previous work, graph drawing systems should also provide the clustering information if the structure of the graph to be drawn is known.

Keywords: Graph drawing aesthetics, clustered graph, visualization, user-sketched layout.

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1. Introduction

For about the past 30 years, most of the graph drawing algorithms have been developed to automatically produce graph layouts in an aesthetically pleasing way, i.e., to optimize the predefined aesthetic criteria as much as possible. Aesthetic criteria in graph drawing indicate graphic structures and properties of graph layouts, such as minimizing number of edge crossings or bends, maximizing number of symmetries, minimizing the drawing area, keeping uniform edge lengths, among others. However, it is generally NP-hard to simultaneously optimize those criteria, in many cases, e.g., force-directed graph drawing algorithm for general graphs [8, 4, 11, 14, 10] and clustered graphs [9, 6, 5]. These problems have been studied extensively in the literature [19, 9]. In those automatic graph drawing algorithms, the aesthetic criteria to be optimized are typically determined from the algorithm designers' points of view, not how the user lays out graphs.

In recent years, a great deal of empirical research has focused on whether aesthetics of the drawings generated by automatic graph drawing algorithms can really help users' understanding, e.g., minimizing number of edge crossings, minimizing number of edge bends, and maximizing the included angles [20, 16, 27, 13]. However, these works mostly addressed the ability of users in reading and interpreting an already presented graph layout, produced by an automatic graph drawing algorithm. Hence, another line of the empirical research is to investigate the graph layouts sketched manually by users, e.g., the work in [17] concluded that edge crossings and grid aligning are considered as the most important factors among user-sketched graph drawing aesthetics.

Clustered graphs could provide users more insights into organization of complicated graphs. Hence, it has been of much interest to investigate how to draw clustered graphs "nicely" [2]. This paper extends the works in [17, 1, 21] to further investigate the aesthetics specifically for user-sketched layouts of clustered graphs with known clustering information. Note that the case without any clustering information has been studied implicitly in [21], so this work further realizes user-sketched clustered graph drawings when their clustering information is provided initially. To achieve this goal, during the experiments, aside from the adjacency list of the experimental clustered graph, we additionally provide the participants the clustering information of the clustered graph, i.e., the information on which cluster each vertex belongs to is known. The participants were asked to sketch the clustered graph as "nicely" as possible based on the given adjacency list and the clustering information.

Note that graph clustering problems (i.e., determining how to classify vertices into clusters) with and without known number of clusters are different, and both problems have been widely studied. Generally, the graph clustering problem with known number of clusters is regarded as being easier than the other problem. This inspires us to investigate whether providing more information on graph clustering can help users sketch clustered graphs. However, only providing number of clusters is still too difficult for users to sketch graphs from scratch. Hence, this work provides the information on clustering partition of vertices, and intends to realize whether users are able to sketch clustered graphs "nicely" (not only to classify vertices in visualization) with this information.

The experimental design of this work is based on the work in [21], which compared differences between the layouts for two graphs produced by all participants; and compared the effects of different participant backgrounds. Hence, after obtaining those user-sketched clustered graph layouts and observing their drawing processes, we intend to explore importance of particular aesthetic criteria which might reveal visual information that has not previously been defined yet. That is, characteristics of the sketched clustered graph drawings and importance of representing cluster structures might be found to provide more direct insight into how users organize vertices in representing clustered graph drawings. Additionally, qualitative rating task was designed to visually assess goodness of a clustering structure in respect to some aesthetic criteria specifically for clustered graphs. For each resultant drawing, we rate the quality from one to five on Likert scale.

The rest of the paper is organized as follows. In Section 2, the main related works are surveyed, and then, the two main previous works related to this work are presented in more detail. Section 3 gives the details of our experiments, including equipment, task, participant, procedure, and the overall experimental process. Then, experimental results for for graph layouts and drawing process are analyzed in Section 4 and Section 5, respectively. Section 6 applies a ranking task by the experimenter to validates the experimental results and analyzes the participants' background. Finally, conclusions and future works are given in Section 7.

2. Related Work

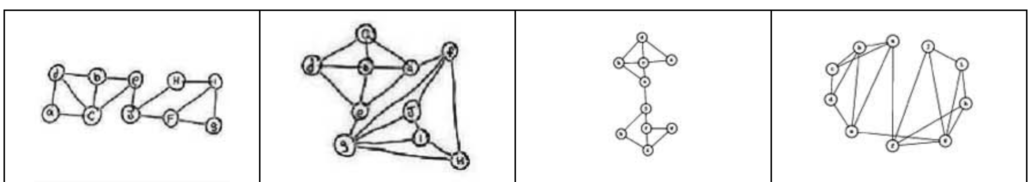
This section gives a preliminary introduction to the related works. First, the relevant works on empirical graph drawing aesthetics are reviewed. Next, as our work is built on the two previous works proposed by Purchase et al. [22, 21], the results in these two works are reviewed in more detail.

Various previous works have been conducted in attempt to produce visually pleasing and easy-to-read graph drawings. For example, one of the earliest works was conducted on readability of diagrams in automatic graph drawings [24]. Huang et al. [10] believed that graph drawings would become more readable if many aesthetics can be satisfied at the same time. Some works focused on creation of graph drawing, e.g., manipulating vertices and edges in a presented layout [25], laying out a graph in a two different layout conditions [7], and so on.

The empirical research done by van Ham and Rogowitz [25] has considered creation of graph layouts. The task in their work is to organize vertices in force-directed graph layouts and circular graph layouts. They used four graphs, each of which has 16 vertices and 2 clusters, respectively. Clusters in their data are connected by one, two, three, four masking edges, respectively. The aesthetic criteria that have been evaluated in their work includes sensitivity of users in representing cluster structures, as well as capability of users in identifying and separating clusters. Note that users' sensitivity in [25] reflects the degree of how users is sensitive to the existence of clusters, and is measured by varying the degree to which clusters were masked by external edges. The other generally-accepted aesthetic criteria include tolerance of users in edge crossings. In their experiment, given an initial graph layout, each participant was asked to reorganize the layout to produce a better graph layout.

One limitation of the existing works is that they have little knowledge about participants' backgrounds. For example, the experiment in [25] was performed online via a website [26], rather than face-to-face. Hence, the authors also claimed that some sophistication existed in their data. The work by Dwyer et al. [7] considered another approach by asking participates to lay out the graph by themselves and comparing performance between user-generated and automatic graph layouts. Additionally, they investigated how participants lay out graphs so as to be appropriate for specific tasks. However, all of those empirical works presented participants with initial layouts of the experimental graphs, so that participants may be biased by the initial layouts.

The empirical works conducted by Purchase et al. in [22, 21] took a very different approach to determining the underlying aesthetics for graph drawing. Participants had to start to draw graphs from scratch, without any initial layouts of the graphs. The only information provided to participants is adjacency lists of the graphs, and their task is to draw easy-to-understand drawings. Each participate should complete two different graph drawings for two respective graphs on a tablet PC with a stylus. In [22], 17 participants were asked to use a graph-drawing software to sketch two graphs, so there are 34 drawings in total. In [21], 34 participants were asked to draw the same graph using two different modes of drawing interfaces: sketch mode (e.g., see Figures 1(a) and 1(b)) and formal mode (e.g., see Figures 1(c) and 1(d)).



(a) Sketch mode for Graph A	(b) Sketch mode for Graph B	(c) Formal mode for Graph A	(d) Formal mode for Graph B

Figure 1: Some experimental results in [21].

Since graph drawing aesthetics are subjective, most previous works on evaluating aesthetic criteria only aimed at the final output drawings. The process of creating graph layouts should also be emphasized and supported, and hence, in [22, 21], effects of aesthetics were considered based on layout products, layout creation processes, and user preferences. Their results found that removing number of edge crossings is the most important aesthetic criterion; while there is some evidence that importance of grid aligning also contributes to graph drawing aesthetics. Some experimental results in [21] are given in Figure 1, including two modes (sketch and formal modes) of two graphs (Graphs A and B). Note that in their design, both the two graphs are clustered graphs (i.e., the vertices in the same cluster are connected strongly): Graph A includes two clusters separated by one bridge edge; Graph B includes two clusters separated by two bridge edges. Although the percentage of visible clusters in each case of the drawings is 58% – 70% in their experimental results, appearance of two clusters is not obvious enough, e.g., see Figure 1. Hence, this inspires the work in this paper that explicitly provides the information on cluster partition to increase possibility of visible clusters.

Note that a clustered graph is defined as sets of vertices and edges, which can be represented as an adjacency list; while a clustered graph drawing is the geometric representation of a clustered graph. In general, clustered graph drawings are divided into two categories: 1) vertices of the same cluster are circled by closed curves (e.g., [3, 28]); 2) the clustered graph is drawn as a general graph drawing in which vertices of the same cluster are drawn closer, while two vertices of different clusters are drawn depart (e.g., [15, 12]). The clustered graph drawings concerned in this paper belong to the second category.

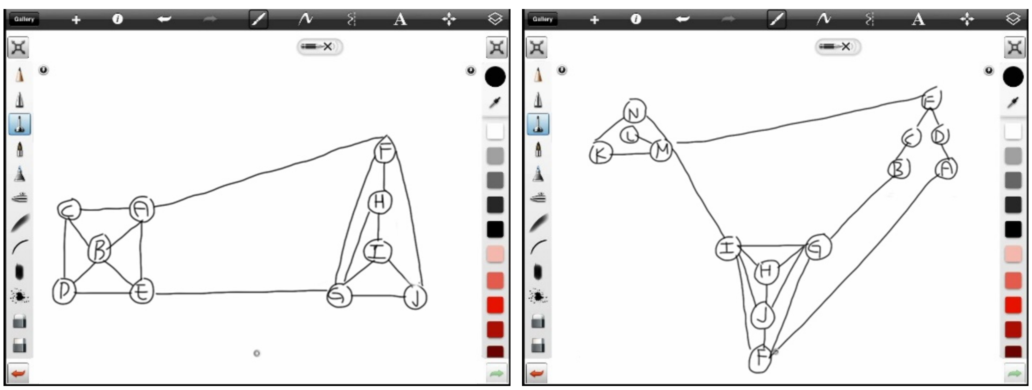
Purchase in [23] revisited and provided a reflective critique of her previous works in [22, 21]. She responded to three questions suggested at two seminars: 1) it seems that the participants compromised their drawing layouts and were not satisfied with the results; 2) it seems that the participants preferred grid-based layouts; 3) it seems that the given adjacency list format influenced the experimental results. Hence, she called back a part of the participants to validate the results done by themselves, and surveyed their satisfaction with a scale of 1–5. Results showed that compromise, preference to grid-based layouts, and graph format influence are not obvious.

3. Our Graph Drawing Experiment

The research question is as follows: *Which graph drawing aesthetics do people favor when sketching their own drawings of clustered graphs with known clustering information?* The question is analyzed by providing participants the adjacency list of a clustered graph and its clustering information, and then asking them to sketch the clustered graph. To analyze the common aesthetics of those drawings, aside from final graph layouts, video of the drawing processes were recorded to analyze the tendency for some aesthetics not presented in final layouts. Additionally, post-interview of participants was conducted to investigate whether their preferences support our speculation.

3.1. Equipment

The hardware used in the experiments is the iPad 2 (a common tablet system), with iOS 6, 64 GB storage, Wi-Fi + 3G model, and a 9.50 inch × 7.31 inch screen. Three iPads with the same OS version and specification were used in the graph drawing tasks. The software that participants used to sketch graphs in the experiments is a simple graph drawing system – Autodesk SketchBook Pro (as shown in Figure 2), which is a popular app on the iPad 2. This app is built with an easy-to-use interface, allowing almost the same hand operations as pen and paper, i.e., any shapes and sizes of vertices and edges can be created, moved, resized, and deleted arbitrarily. For the used stylus, as the capacitive screen of the iPad 2 requires a certain amount of surface area to be covered to respond to touch, and thus, the precision disc of Jot Touch Stylus (nearly identical to a 0.05 mm mechanical pencil or a fine ballpoint pen) was used in the experiments, allowing participants with an even limited experience to be able to pursue their tasks naturally.



(a) (b)

Figure 2: Illustration of the Sketchbook Pro System. The drawings of (a) Graph A and (b) Graph B sketched by Participant #21.

Although the drawings produced by the sketch mode in the system in [21] have many similar features with the drawings produced by the system used in this paper, they differ as follows. The system used in this paper is a generic drawing system, not specifically designed for graph drawing; while the work in [21] developed a graph drawing system, in which, when a vertex moves, the edges associated with this vertex move as well.

All activities on the screen during the whole drawing process were recorded by a screen casting app, Display Recorder (version 1.0.0), which includes response time, audio and visual activities, and interaction with SketchBook Pro. Since video was built in a web server and able to be download in a relatively short period of time, the experiments were conducted in a WiFi environment.

3.2. Task

The task in this paper was to provide each participant the adjacency lists of two experimental clustered graphs and their clustering information, and ask him/her to draw the two graphs with the Sketchbook Pro app on a tablet system using a stylus with the following instruction: *"Please draw the clustered graph in the best way that you think to reflect its clusters and interconnections."* Except that the notations of the adjacency list of edges and the clustering information were explained before the task, the participants were not prompted with any additional information that characterizes readability of graph drawing, such as minimizing edge crossing, minimizing number of bends, etc. The ordering of edges in each adjacency list was random for each participant to prevent the biased case where all participants follow the same ordering to have a similar drawing tendency.

Note that we keep the question as simple as possible, to make the participants be able to be objective to sketch their graphs based on their minds. This concept follows the work in [21].

3.3. Experimental Graphs

Each participant was asked to draw two graphs: Graph A with 10 vertices and 18 edges; Graph B with 14 vertices and 21 edges. For convenience of realizing structures of the two graphs, we can refer the drawings of the two graphs (sketched by Participate #21) in Figure 2. From the two drawings, it is observed that Graph A and Graph B can be divided into 2 and 3 clusters, respectively.

The experiments provided participants only the adjacency lists of the two experimental graphs and their clustering information as illustrated in Table 1, in which each of the two clusters $\mathcal{C}_1$ and $\mathcal{C}_2$ in Graph A includes 5 vertices; the three clusters $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{C}_3$ in Graph B include 4, 5, and 5 vertices, respectively. As explained in [21], the experimental graphs used in the user-sketched graph drawing studies are smaller to those in [25, 7], because the user-sketched graph drawing task is more difficult and we hope to keep the experimental duration within reasonable time.

Note that the provided information is both intrinsic and extrinsic. It is intrinsic because the experimental graphs are originated or tailored from those in [23] in which only one or two bridge edges exist in the experimental graphs, i.e., two clusters exist obviously. It is extrinsic because we provided the predefined clustering information (i.e., Table 1) to the participants.

Table 1: Adjacency lists and cluster partitions of the two experimental graphs.

Graph A	
Adjacency list	(A,B) (A,C) (A,E) (A,F) (B,C) (B,D) (B,E) (C,D) (D,E) (E,G) (F,G) (F,H) (F,J) (G,H) (G,I) (G,J) (H,I) (I,J)
Clusters	$\mathcal{C}_1 = \{$A, B, C, D, E$\}$, $\mathcal{C}_2 = \{$G, H, I, J, F$\}$
Graph B	
Adjacency list	(A,D) (A,F) (B,C) (B,G) (C,E) (D,E) (E,M) (F,I) (F,J) (F,G) (G,H) (G,I) (G,J) (H,I) (H,J) (I,J) (I,M) (K,M) (K,N) (L,M) (M,N)
Clusters	$\mathcal{C}_1 = \{$K, L, M, N$\}$, $\mathcal{C}_2 = \{$F, G, H, I, J$\}$, $\mathcal{C}_3 = \{$A, B, C, D, E$\}$

Design of our two experimental graphs are explained as follows. Our Graph A is the same as the Graph B in [21], except that we provide its clustering information. By doing so, performance of our resultant drawings for Graph A can be compared with [21]. As for our Graph B, we intend to analyze how participants draw a graph with more clusters, and therefore, Graph B includes three clusters with different styles connected by some bridge edges (Figure 2(b)).

Note that it is difficult for participants to sketch larger graphs manually. However, since the whole experiment conducted in this work took about one hour, we did not consider the task for larger graphs, because human patience and attention to finishing such a complex task is limited. In addition, most previous related works (e.g., [25, 7]) also conducted experiments on smaller graphs. By doing so, participants could focus more on increasing the quality of their drawings, so that the resultant drawings are simpler to be analyzed.

3.4. Participants

We recruited 30 participants in the experiments, who had normal or correct-to-normal vision. Most of them are the students from National Chiao Tung University (NCTU) studying different fields of engineering, while the pool also include a small number of friends, family, and colleagues. The demographic results are listed in Table 2, in which the average age of participants was 27, ranging from 18 to 36; at the time of their participation, 17 of them indicated that they had no prior experience related to graphs, while the others mentioned that they had fundamental ideas about graphs.

Table 2: Demographic information of the participants.

Attribute	Constitution
Gender	13 female, 17 male
Highest education level	6 high school, 11 bachelor, 13 master or Ph.D. degree
Age	18–36 years old
Current occupation	21 students, 9 non-students
Experience in using touch screen devices (e.g., tablet PC, smart phone)	23 yes, 7 no
Experience in graphs	13 yes, 17 no
Experience in using a stylus	7 yes, 23 no

3.5. Experimental Process

The experiments were conducted in a meeting room in NCTU. Before starting the experiments, each participant was asked to read an information sheet, understand the task, sign a consent form, complete a paper-based questionnaire which inquired his/her demographic information, as well as personal experience in graphs and digital drawings using a stylus. After filling the questionnaire, to ensure participants to be comfortable with the graph drawing task and the system procedure, they were then given a short tutorial of five minutes which demonstrates the SketchBook Pro System, including how to navigate the function menu interface and set a bunch of preferences: sketching, erasing, relocating, selecting, width setting, undoing, redoing action, zooming and scrolling by tapping menu icons on the top of the screen.

In the tutorial, definition of clustered graph drawings (without circling vertices of the same cluster by using closed curves) are explained, and hence, no participants represented the clusters by using closed curves.

After the tutorial, to make sure whether participants can get familiar with the task in the system before the real experiments, participants were given an opportunity to draw two simple trial graphs in the system directly. The first trial graph consists of 6 vertices and 6 edges; and the second trial graph consists of 9 vertices and 9 edges. Each of the two trial graphs can be divided into 2 clusters. Note that they were not informed of constitution of the trial graphs.

Once ready to start, participants started to perform the drawing task formally. To avoid that ordering of edge pairs could affect the experimental results, ordering of the two end vertices of each edge in each provided adjacency list is random, and ordering of edges in the adjacency list is randomized. Additionally, the first graph to be drawn is random, so that 13 of the 30 participants drew Graph B before Graph A in the real experiments. From the pre-trial experiments, some participants indicated that drawing two graphs without a short break might cause disturbances on vision. As a result, during the experiments, participants were given an 8 minute compulsory break before drawing the second graph.

During the experiments, there was no time limit, and hence participants could take their time as long as they wanted. It turns out that the trial experiments took about 23 minutes on average; the whole experiment took about 65 minutes on average, including the preparation time, demonstration, debriefing, and breaks.

4. Experimental Results for Graph Layouts

Based on the experimental design in the previous sections, this section gives the experimental results and comprehensive discussion. At first, the experimental graph drawing results are given and discussed. Then, characteristics of the drawings, drawing behavior of participants, and the clustered structure of the drawings are analyzed. Finally, the participants' backgrounds are analyzed.

4.1. Graph Drawing Results

Since each of the 30 participants produced two drawings, 60 drawings were produced in the experiments (Figures 3 and 4). For notational convenience, the drawings of Graph A and Graph B produced by participant # 1 are denoted as 1A and 1B, respectively; while the other drawings are denoted in the same way.

4.2. Characteristics of Clustered Graph Drawings

Aesthetics of a graph are graphic structures and properties of a "nice" drawing of the graph. The following characteristics are visually analyzed for evaluating aesthetics of the graph drawings [22, 21]:

- Human error: The total number of incorrect drawings is counted.
- Number of edge crossings: An edge crossing means a pair of edges that intersect each other. Conventionally, a "nice" drawing contains fewer edge crossings; while some previous works (e.g., [10]) proposed that a drawing with larger crossing angles would also be "nice".
- Number of bends: A bend is a point on an edge which changes its direction. Fewer bends are preferred conventionally. Numbers of bends found in intra-cluster edges and inter-cluster edges are counted, respectively.
- Number of orthogonal-like drawings: In an orthogonal-like drawing, the graph is drawn on a plane and most of the edges are represented by alternating sequences of horizontal and vertical segments.

Aside from the above criteria used in common empirical studies, we additionally consider the following measure for clustered graph drawings:

- Number and type of local symmetries: A drawing has a *reflectional symmetry* if the drawing can be folded in half along a reflectional axis and the two halves line up with each, while the drawing has a *k-rotational symmetry* if it can be rotated around degrees $2\pi/k$ and still look the same. In general, a "nice" drawing contains more symmetries in global or local visualization. In the plane drawing of Graph A (Figure 2(a)), the drawing of the cluster consisting of vertices A-C has reflectional and 4-rotational symmetries, while the other one excluding edge GH has a reflectional symmetry. In the plane drawing of Graph B (Figure 2(b)), the drawing of the cluster consisting of A-E is a chain which can be drawn symmetrically, the one consisting of F-J has a reflectional symmetry, and the other one excluding edge LM is a

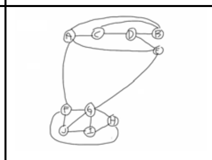
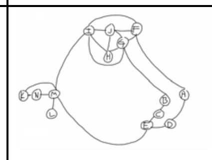
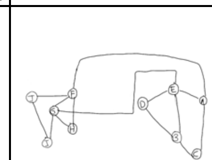
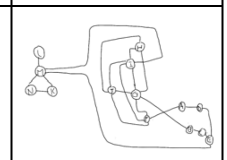
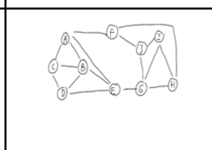
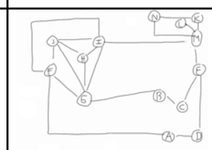
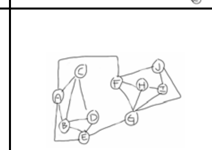
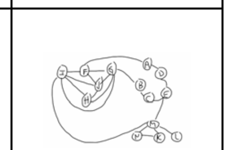
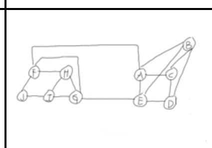
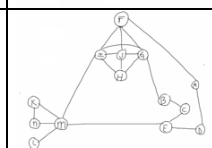
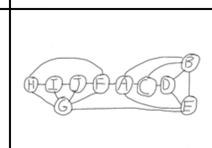
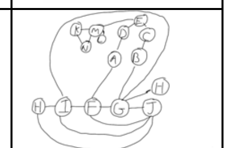
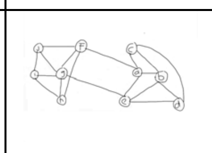
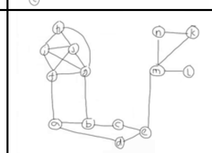
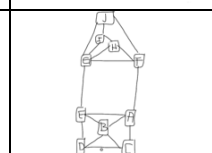
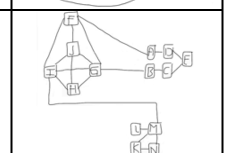
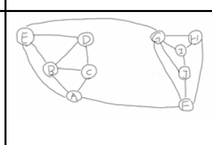
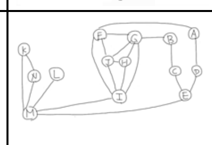
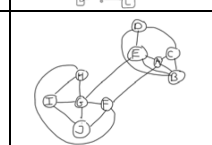
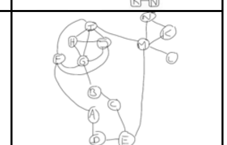
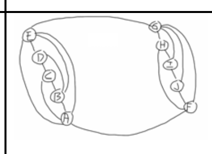
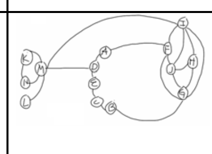
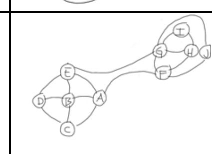
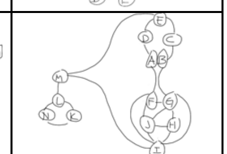
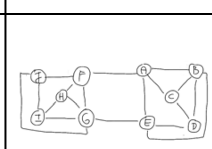
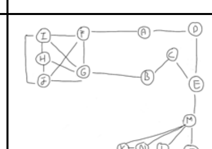
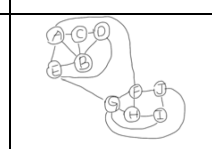
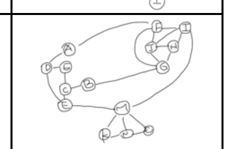
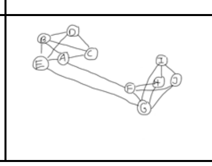
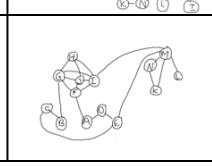
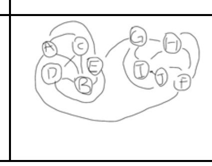
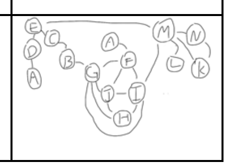
Parti- -cipant	Graph A	Graph B	Parti- -cipant	Graph A	Graph B
1			**9**		
2			**10**		
3			**11**		
4			**12**		
5			**13**		
6			**14**		
7			**15**		
8			**16**		

Figure 3: Resultant drawings drawn by participants #1 – #16.

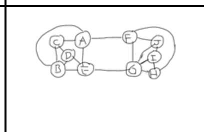
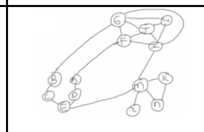
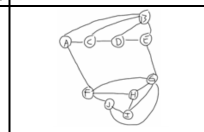
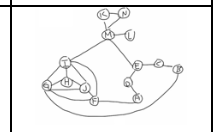
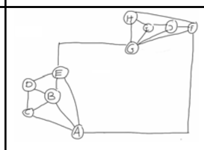
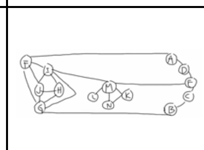
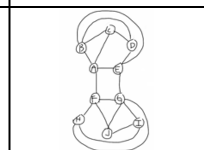
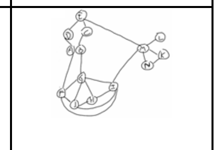
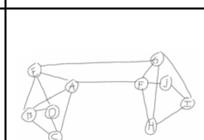
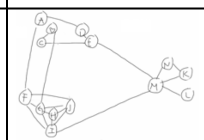
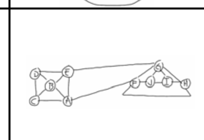
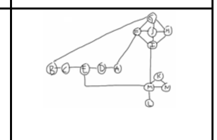
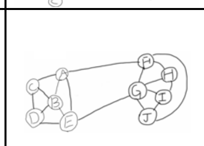
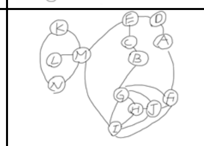
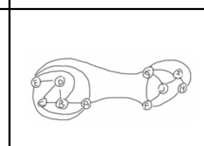
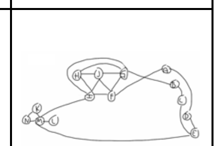
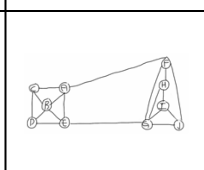
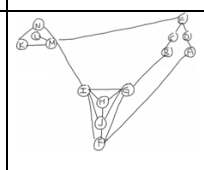
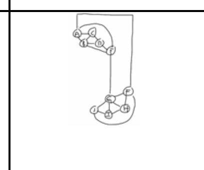
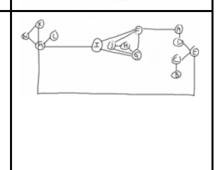
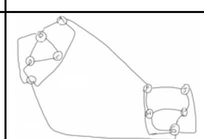
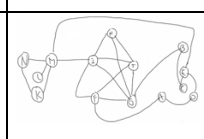
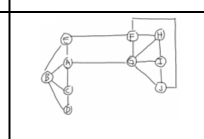
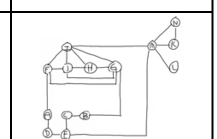
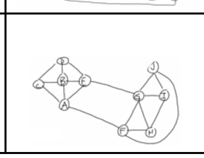
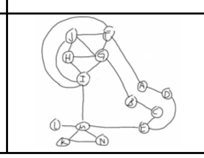
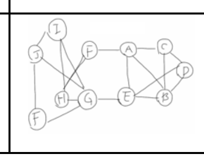
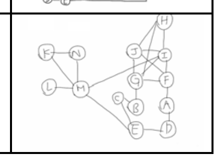
Parti- -cipant	Graph A	Graph B	Parti- -cipant	Graph A	Graph B
17			**24**		
18			**25**		
19			**26**		
20			**27**		
21			**28**		
22			**29**		
23			**30**		

Figure 4: Resultant drawings drawn by participants #17 – #30.

triangle, allowing reflectional and 3-rotational symmetries. Since each cluster, probably excluding some edge, in our experimental graphs may be symmetric but the whole graph is not, we call such a drawing to be locally symmetric.

Note that this paper is concerned with only the visual analysis on evaluating aesthetic criteria, i.e., algorithm design and computational analysis are beyond our main concern.

In what follows, we analyze each of those characteristics in detail.

4.2.1. Human Error

Since the experiments were conducted in a tightly controlled environment and a sophisticated procedure was applied, participants are unlikely to create totally-flawless drawings. Based on the resultant drawings, we found that the errors that appeared most frequently were very similar, and were categorized into missing edges, additional edges (i.e., some edge was drawn twice), and additional vertices. Summary of the incorrect drawings is presented in Table 3, in which for Graph A, there are 7 drawings with missing edges, 2 drawings with additional edges, and none with additional vertices; for Graph B, there are 6 drawings with missing edges, 3 drawings with additional edges, and 3 drawings with additional vertices. Majority of the errors in the drawings come from missing edges, which account for around 21.6%. The errors caused by additional edges account for approximately 8.3%, and only 3/60 (5%) drawings have additional vertices.

Table 3: Summary of incorrect drawings.

	Graph A	Graph B
Missing edge	1A (BE)	4B (IM)
	2A (GF)	6B (EM)
	3A (GJ)	11B (MI,GI,ME)
	9A (CD,BA,GJ,HI)	15B (GI)
	19A (GJ)	26B (GI)
	22A (AC)	28B (JI, HI, GB)
	29A (ED)	
Additional edge	6A (EA)	4B (AB)
	27A (EA)	6B (DM)
		15B (NL)
Additional vertex		7B (I)
		11B (H)
		15B (E)

Note that the main concern in the experiments is to analyze the behavior of how the clustered graph was expressed and created, and hence, we do not make any judgment to the participants if their drawings are not correct. Note that the experimental results in previous works (e.g., [22, 21]) also included incorrect drawings.

4.2.2. Edge Crossing

The statistics on edge crossings are recorded in Table 4. The number of edge crossings for Graph B is greater than that for Graph A. Among the 60 resultant drawings, there are 9 drawings with only one crossing, 5 drawings with two crossings, 2 drawings with three crossings, 2 drawings with five crossings, and 1 drawing with six crossings. The ratio of the drawings with edge crossings to all the drawings was approximately 31.6%.

Since our Graph A is the same with the Graph B in the work of [21], it is of interest to make a comparison between our experimental results and theirs. The total number of edge crossings for our Graph A is 18; while the numbers for the Graph B in [21] are 51 and 39 in sketch and formal modes, respectively. As for the maximum number of edge crossings, the numbers for the Graph B in [21] are 22 and 18 in sketch and formal modes, respectively, both of which are much greater than our Graph A (only 5). It can be concluded that providing the clustering information can help considerably reduce the total and the maximum numbers of edge crossings, which is surprising because this is not the original purpose of providing the information in this work. We further speculate that when participants were asked to sketch graphs in a pleasing way based on the known cluster information, they would pay much attention to following this information, so that they tried to avoid any edge crossings when laying out vertices and edges one by one. Furthermore, although our Graph B has more vertices and edges than Graph A, the measures between the two graphs have not many differences from the p-value in Table 4. It implies that providing the clustering information can help reduce number of edge crossings, even if the graph size becomes larger.

Note that mean percentage of edge crossings is calculated with the same formula in [21], which computes the number of edge crossings over the approximated upper bound on number of edge crossings in [18] for all drawings, and then takes their mean.

4.2.3. Bend

Different from [22, 21] that measured number of straight edges, we measure number of bends on the edges within clusters (a.k.a., intra-cluster bend) and the edges between clusters (a.k.a., inter-cluster bend). The mean percentage of our resultant drawings with bends is 45%, in which 25% of the

Table 4: Statistics for different aesthetics measurements.

	Graph A	Graph B	p-value
Edge crossing			
Number of edge crossings	18	23	0.632
Mean	0.600	0.766	
Median	1.5	2	
Maximum	5	6	
Standard deviation	1.32	1.38	
Mean percentage of edge crossings	0.57%	0.49%	
Number of plane drawings	22	19	0.413
Bend			
Number of inter-cluster edge turns	16	17	0.919
Mean	0.53	0.56	
Median	2	1.5	
Maximum	5	7	
Standard deviation	1.13	1.38	
Number of intra-cluster edge turns	21	23	0.894
Mean	0.70	0.76	
Median	2.5	3	
Maximum	6	11	
Standard deviation	1.64	2.20	
Orthogonal-like drawing			
Number of orthogonal-like drawings	11	10	0.599
Mean	0.36	0.34	
Standard deviation	0.49	0.47	
Variance	0.24	0.23	
Local symmetry			
Number of symmetries	11	11	1.000
Number of reflectional symmetries	7	9	0.621
Number of rotational symmetries	4	2	0.398
Mean percentage of local symmetrical drawings	26.67%	26.67%	
Maximum	2	2	
Standard deviation	0.66	0.66	

graphs were drawn with inter-cluster bends; while 20% were drawn with intra-cluster bends. 7 drawings has 1 bend, 12 drawings has 2 bends, and 8 drawings with 7 bends. The number of bends in the experiments is greater than that for the sketch mode in [22, 21], in which most drawings are straight-line. We think it reasonable because participants attempted to classify vertices into clusters and hence make vertices of different clusters apart so that vertices of the same cluster are needed to be drawn in a smaller restricted drawing area, which leads to more bent edges.

The comprehensive results of bends between Graph A and Graph B are shown in Table 4, in which Graph B with more edges unsurprisingly has more bends than Graph A.

4.2.4. Orthogonal-like Drawing

As mentioned in the above discussion on number of bends, to depart vertices of different clusters so as to draw vertices of the same cluster in a smaller restricted drawing area, the participants applied bent edges, in which some participants preferred orthogonal edges because fewer line directions are employed: 35% of our experimental drawings are orthogonal-like, e.g., Drawing 29B in Figure 4 is a pleasing orthogonal-like drawing; while Drawing 12A in Figure 3 has a tidy upright representation. From Table 4, the statistics on orthogonal-like drawings between Graph A and Graph B do not differ a lot. Additionally, we observe that the participants who drew the first graph orthogonally tended to draw the second graph orthogonally. About 10 participants drew both graphs orthogonally.

4.2.5. Local symmetry

Of the 60 drawing produced in the experiments, 16 drawings are locally symmetric, in which 10 drawings (i.e., 12A, 13A, 14A, 29A, 3B, 9B, 12B, 14B, 20B, 21B) have locally reflectional symmetries, 2 drawings (21A, 15B) have locally rotational symmetries, and 4 drawings have both (i.e., 20A, 23A, 26A, 26B). Among them, 8 drawings for Graph A account for 26.67% of all 30 drawings; those for Graph B have the same ratio. From Table 4, the total number of symmetries is 22: 16 reflectional and 6 rotational symmetries, respectively.

5. Experimental Results for Drawing Process

To investigate how the graph layouts were created, what kind of layout skills the participants had applied, how the drawing behavior influenced the resultant layout product, and the influence of the creation time through the final layout product. Therefore, this subsection studies the screen cast video to perform more comprehensive analysis.

5.1. Creation Time Data

All creation times were recorded and standardized using the box and whisker diagram (Figure 5), in which distributions of the creation times for Graph A, Graph B, and both graphs are displayed based on their upper quartile, lower quartile, inter-quartile ranges, and mean. With more clusters, vertices and edges, the task for drawing graph B is literally more challenging as compared to drawing graph A, and hence might consume more time. On average, each participant consumed more or less than 12 minutes for drawing Graph B; Graph A was drawn with approximately 8 minutes (Figure 5). Analysis from the display recorder showed that as the drawing process becomes more complicated, some of the participants tended to make unintended actions such as losing an edge, creating an extra edge, and adding redundant vertices. As structure of Graph B is more complicated, mistakes for Graph B happened more frequently than those for Graph A.

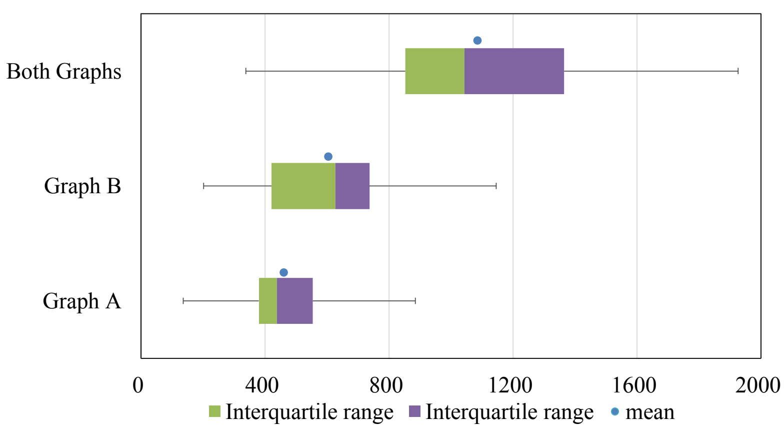


Figure 5: Box and whisker diagram displaying the range and the mean of the creation time (in seconds).

5.2. Drawing Strategy and Preference

The drawing strategies that were preferred most commonly by the participants during the graph drawing process are summarized as below:

- Simply placing all vertices first and inserting edges later on.
- Placing vertices and edges wherever convenient.
- Drawing vertices and edges one by one.
- Relocating some vertices to ease the later edge insertion and avoid possible crossings.
- Favoring bent, curved, and orthogonal edges.
- Focusing on finishing drawing one cluster before moving focus to the next cluster.
- Inserting inter-cluster edges (bridge edges) after all clustered graphs are drawn.
- Planning positions of all clusters before starting the task.
- Planning ordering of vertices before starting the task.
- Maintaining uniform edge length, and vertices of the same cluster with high cohesion.

A comprehensive analysis for the drawing strategies including participants' preferences and number of participants is summarized in Table 5. Consider the strategies that the participants preferred for either drawing all vertices first or drawing vertices or edges simultaneously (i.e., see the 'For vertices' categories in Table 5). We have the following speculation:

- Most participants preferred drawing vertices and edges simultaneously (see the 'Drawing vertices and edges simultaneously' sub-category in Table 5). 22 participants preferred the one-by-one strategy which allows more precise pace in identifying vertices and also producing drawings more efficiently; 14 participants revealed the necessity to relocate vertices to ease the later edge insertion and prevent possible crossings.
- With only a few participants who preferred to place all vertices first (see the 'Placing vertices first' sub-category in Table 5), we speculate that those participants would like to have a clear picture of how to group vertices to form a cluster first. Eight participants attempted to place some vertices first and then insert edges whenever convenient, as

this strategy was more convenient and more casual to be applied during the drawing process.

Table 5: Different drawing strategies preferred by 30 participants.

Strategy	Number of participants	Explanation
For vertices		
Placing vertices first		
Placing all vertices first according to their clustering information	6	3 for both graphs; 2 for Graph A; 1 for Graph B
Placing some vertices and inserting edges wherever convenient	8	2 for both graphs; 3 for Graph A; 3 for Graph B
Drawing vertices and edges simultaneously		
Drawing vertices and inserting the next edges one by one	22	14 for both graphs; 5 for Graph A; 3 for Graph B
Relocating vertices to ease the later edge insertion and avoid crossings	14	4 for both graphs; 4 for Graph A; 6 for Graph B
For edges		
Favoring bent edges	15	8 for both graphs; 3 for Graph A; 4 for Graph B
Favoring curved edges	19	15 for both graphs; 1 for Graph A; 3 for Graph B
Favoring orthogonal edges	15	12 for both graphs; 3 for Graph A
For clusters		
Focusing on drawing a single cluster	23	19 for both graphs; 3 for Graph A; 1 for Graph B
Inserting inter-cluster edges after all clusters are drawn	14	9 for both graphs; 3 for Graph A; 2 for Graph B
For the planning process		
Planning ordering of drawing vertices to avoid crossings	20	This was observed by repeated pausing actions and changing vertices during the drawing process
Planning positions of clusters	13	9 for both graphs; 4 for Graph A
Maintaining each cluster with a similar size	27	17 for both graphs; 9 for Graph A; 1 for Graph B
Maintaining uniform edge length	22	22 for both graphs
Drawing vertices in the same cluster with high cohesion	25	Vertices in the same cluster are strongly-connected in their graph layouts.

From the 'For edges' category in Table 5, half participants preferred to have bent and orthogonal edges in their drawings, as vertical and horizontal lines were intended to present a linear sense and good visual drawing perspective; two thirds of the participants favored curved edges as it appeared to have a smooth and natural sense on the edge lines.

When considering clustered graphs that best reflect their interconnections (see the 'For clusters' category in Table 5), 23 participants paid attention to drawing one cluster before drawing the other clusters. We speculate that they were afraid of losing focus and getting distracted. To highlight the relationship among other clusters, 14 participants preferred creating inter-cluster edges after the corresponding clusters were obviously drawn.

Analyzing the display recorder video, it is observed that most of the participants had a planning process in order to create the best clustered graph drawing based on their minds. The observations are stated as follows:

- The planning behavior was observed by repeated pausing actions during the drawing process, especially for removing edge crossings (with 20 participants from Table 5).
- From pausing actions for planning cluster structures and sizes, some participants (13 participants from Table 5) were likely to arrange positions of clusters. Majority of the participants (27 participants from Table 5) had much interest in maintaining each cluster with a similar size in the graph drawing.
- Edges and vertices were frequently removed and recreated within a cluster, so as to put an emphasis on maintaining a similar edge length (with 22 participants from Table 5).
- During the drawing process, the orderings of vertices applied by most participants (25 participants from Table 5) is decided to exhibit tightly-connected vertices of the same cluster, which results in a drawing of the cluster with high cohesion.

6. Validation and Demographic Analysis

This section applies a ranking task by the experimenter to validates the experimental results, statistically analyzes the experimental results, and analyzes the participants' background.

6.1. Performance of Clustered Graph Drawing in Experimental Results

To evaluate the clustering performance of the resultant drawings, we conducted a rating task using the five point Likert scale (from strongly agree to strongly disagree) to answer the following questions for each graph:

- Importance of bridges: Some edges are important because they form bridges between clusters. How would you rate the graph with respect to the importance they appear to be as bridges between clusters?

- Cluster cohesiveness: Cluster with cohesiveness would increase the feeling of the cluster as a whole. As a result, cluster acts as a whole, not as individual, and vertices of the same cluster tend to have strong interactions with one another. How would you rate the graph with respect to cohesiveness that provides the bonds to hold the vertices in a cluster together?

Statistics for rating task measurement are given in Table 6, from which most participants 'strongly agreed' or 'agreed' importance of bridges and cluster cohesiveness.

Table 6: The rating task in the questionnaire.

Question	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
1. Importance of bridges (Graph A)	53%	27%	13%	7%	0%
2. Importance of bridges (Graph B)	27%	37%	30%	3%	3%
3. Cluster cohesiveness (Graph A)	27%	47%	20%	3%	3%
4. Cluster cohesiveness (Graph B)	23%	30%	40%	3%	3%

6.2. Statistical Discussion of Aesthetics Measurements

Table 4 shows the statistical analysis for different aesthetics measurements in drawings of Graphs A and B produced by all participants. To examine the relationship between drawings of the two graphs, we gather all aesthetics data from all graph drawings, and then perform independent t-test, using a comprehensive statistical t-test program offered by STATISTICA, to obtain significant values. This significant value (p-value) is defined as the probability of significance that indicates whether the differences between two graph drawings most likely reflect a real difference in the graph drawings from which the data are sampled.

The p-values are displayed in the last column of Table 4. Using the standard alpha level of 0.05, we find that none of our results is significant. This is a very surprising result, as the design of our graph B is more complicated. With these p-value findings, we provide the following support:

- When participants introduce edge crossings in their drawings for Graph A, it does not necessarily imply that edge crossings will also occur in their drawings for Graph B.

- There is no significant relation between numbers of bends in both graph drawings. Producing an intra-cluster bend in the drawing for Graph A will not influence producing an intra-cluster bend in the drawing for Graph B. This relation is also applied for inter-cluster bends.

- The p-values found in numbers of local reflection symmetries ($p = 0.621$) and local rotational symmetries ($p = 0.398$) are heading towards significance. In both graph drawings, the total number of local symmetries is same.

A conclusion can be drawn from our t-test is that both the drawings for Graphs A and B have no significant impact with respect to number of edge crossings, bends, symmetries, and grid drawings.

6.3. Analysis of Participant Background

We are interested in understanding the participants' skill degrees to organize vertices and edges of a graph into an obvious picture that can represent clusters and their interconnections. Therefore, we examine the relationships of the participants' demographic backgrounds and the extent to which the participants explored importance of certain visual characteristics such as minimizing edge crossings, maximizing symmetries, adopting uniform edge lengths, laying out orthogonal-like drawings, maintaining each cluster with a similar size, and other aesthetic criteria.

The basic hypothesis in this analysis is that the participants who had graph drawing experiences, either fundamental ideas about graphs or related drawing background would be likely to draw the layout that characterizes readability and aesthetics. Table 7 shows the analysis of preferences, visibility, cluster representation, and different drawing strategies between experienced and non-experienced participants. Based on this analysis, the experienced participants are more responsive to some particular features such as uniform length, size of cluster, reduction in bent edges, more symmetrical drawings, and shorter distance between vertices of the same cluster.

Surprisingly the data in Table 7 suggests that the experienced participants tended to draw graphs that maximize symmetries, while the non-experienced participants prefer an alternative strategy to exhibit visibility

Table 7: Analysis of the participants' backgrounds and resultant drawings.

	Experienced participant	Non-experienced participant
Number of participants	13	17
Number of graphs	26	34
Average creation time for both graphs (min)	16.3	17.8
Preferences (over all drawings)		
Percentage of orthogonal edges	26%	33%
Percentage of edge crossings	5%	61%
Percentage of curved edges	28%	30%
Percentage of non-crossing graph drawings	18%	12%
Strong visibility (over all drawings)		
Number of symmetrical graph drawings	15	7
Number of orthogonal graph drawings	10	12
Number of the graph drawings with bent edges	21	56
Importance on representing a cluster structure (over all participants)		
Minimizing the distance between vertices of the same cluster	12	10
Making the vertices of the same cluster highly interconnected	12	13
Drawing each cluster with high cohesiveness	12	10
Importance of representing the bridge edges	11	15
Uniform-length edges	13	9
Drawing each cluster with a similar size	12	8
Planning ahead for cluster position	8	5
Manipulating vertex positions to represent the best cluster relationship	8	4
The drawing strategy (over all participants)		
Placing all vertices according to their clustering information	3	3
Placing some vertices and inserting edges wherever convenient	5	3
Drawing vertices and inserting the next edges one by one	7	15
Relocating vertices to ease the later edge insertion and avoid crossings	8	6
Planning ahead for the vertex insertion order	9	11

such as orthogonal-like and bent edges. Accordingly, these participants had made prediction for their cluster positions and manipulated vertex positions until they felt satisfied with the cluster structures and the relationships. These gave us indication of how their graph drawing attentions were directed to represent clusters and their interconnections.

7. Conclusion and Future Work

By providing the adjacency list of a clustered graph and its clustering information, the main task in this work is to ask participants to create the best graph drawing of a clustered graph that can reflect clusters and their interconnections. We can speculate that most of the participants had performed well in representing their clustered graph drawings. The results have totally addressed many hidden features that were not analyzed by previous works in empirical graph drawing experiments. This paper has investigated the aesthetics of clustered graph drawings created by participants, and the resultant drawing results show more improvements to the overall visual aesthetic quality of graph drawings. The total number of edge crossings found in the resultant drawings is relatively small as compared to the previous works in [22, 21]. Although providing clustering information in the graph drawing task aims to assist participants in clustering vertices, it made significant contribution in human drawing aesthetics from our results, e.g., more pleasant graph drawings with less crossings and more symmetries. Additionally, some special edge styles that increase visibility of the graph drawings are also found, e.g., bent, curved, and orthogonal-like edges. Therefore, it is important to consider various aesthetic aspects and graph structures during the graph drawing process.

The main difference of this work from the previous works is to provide clustering information. With the information, the layout creation has higher possibility to become more readable and fulfill aesthetic extents as many as possible at the same time. Additional measurement is also considered in respect to goodness of the clustering structure. Specifically, we conduct a questionnaire study to evaluate importance of bridges and the cluster cohesiveness. Measuring various aesthetic aspects in graph drawing creation gives us a sense of the extent to which the drawing conforms to participants. In conclusion, these experiments reveal that although participants created different unique layouts, they very reliably converge on cluster representations that share important similarities. They seek to minimize edge crossings, they enclose highly-connected vertices within a cluster, and they are extremely competent at identifying bridges and introducing different drawing strategies that manipulating the vertices so that the best represent the clustered graph relationship.

From the viewpoint on designing the graph drawing systems, the previous work in [21] suggested that the graph drawing system should be equipped with the grid-based editing function, because most of their experimental results show grid-like edges. From our experimental results, we further show that providing the clustering information of a graph can help users draw the graphs with clusters in a pleasing way. Hence, our results support that aside from the grid-based editing function suggested by the previous work, the graph drawing systems should also provide the clustering information if the while structure of the graph to be drawn is known, so as to alleviate the complex drawing task.

The limitation of our graph reading task is that much of our data collection was qualitative and collected by observation, rather than by quantitative computational means. Second, due to the size of the iPad 2, our graph drawing task is limited up to three clusters so that it is difficult to experience more complex graphs. In the future, we would like to conduct more challenging graph drawing experiments with larger graphs, and would consider our concerned clustering problem with different versions of graph reading tasks by providing other aesthetic constraint at the same time, e.g., no bent edges, no crossings, etc. It is also hoped in the future that we can use a quantitative approach to measure the aesthetic criteria. Another line of future work is to ask design artists to conduct experiments.

It is also of interest to discuss whether the participants compromised the aesthetics that they favored as the graph became more complicated, as if the work in [23] discussed this question for the experiments in [21], which this paper is based on.

Acknowledgements

The authors thank the anonymous referees for comments that improved the content as well as the presentation of this paper. This work has been supported in part by MOST 104-2221-E-009-134-MY2, Taiwan.

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