Energy-Efficient Cell Association and Load Analysis in Heterogeneous Cellular Networks

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Abstract—This paper exploits the fundamental connection between cell association and load in the sense of bits-per-joule energy efficiency. Users adopt a proposed energy-efficiency-based association reward function to associate with their base stations in a heterogeneous cellular network consisting of \( K \) independent Poisson point processes (PPPs) of base stations (BSs). We first investigate the cell association probability and the statistics of the \( k \)-th tier BSs for a universal association reward function and they can be tailored for any cell association schemes that satisfy the constraints on the association reward function. Under the proposed energy-efficient cell association (EECA) scheme, the statistical properties of an associated BS, coverage probability, average energy efficiency and cell load are all derived and analyzed. We show that EECA can achieve a (much) higher coverage probability, average energy efficiency, and better cell load balancing if compared with maximum received power and nearest BS association schemes.

I. INTRODUCTION

In recent years, the topology of cellular networks has been gradually migrating from a single type of base stations to multiple types of heterogeneous base stations. Such a heterogeneous cellular network (HetNet) consisting of different kinds of base stations (BSs), such as macro BSs, picocells, femtocells, etc., potentially can carry huge network throughput in order to handle the increasingly large traffic flow due to the proliferations of smart wireless handsets [1]. Although deploying different kinds of small cells overlaid on the tier of macro BSs can significantly increase network throughput, how to effectively utilize the recourses offered by those small cells to compensate the additional cost (e.g. operation and energy cost) due to the small cell deployment is a crucial issue that needs our attention.

Energy-efficient transmission techniques are certainly worthy of exploring in a heavy-traffic HetNet since a large amount of energy can be saved while transmitting. Current energy-saving approaches for cellular networks mainly focus on the BS-level management such as power control, cell activation, scheduling, etc. [2]–[4], and the works based on the concept of energy-efficient traffic management are still minimal. Traffic flow behaviors in a HetNet are significantly affected by the adopted cell association strategies. If there exists an energy-efficient approach to make all the traffic flow into energy-efficient access points, saving large amount of energy can be surely expected. This motivates the idea of the energy-efficient cell association scheme proposed in this paper.

Previous works on cell association and traffic offloading focused on the study of the relationship between cell association, throughput and resource. For example, reference [5] proposed a biased maximum received power association and studied the average throughput of a downlink channel in a HetNet, but the issue between cell association and load was not addressed. The resource allocation, partitioning and traffic offloading problems were investigated in [6]–[8]. The statistics of the associated BS in a particular tier was not studied and thus only the load fraction for each tier was found. None of these prior works studied the problem of cell association and traffic offloading from an energy-efficient perspective. As a result, in general their proposed schemes on cell association and traffic offloading cannot achieve the optimality of energy efficiency.

To achieve the goal of saving energy while directing traffic over a HetNet, an energy-efficient cell association (EECA) scheme for a \( K \)-tier HetNet was proposed in this paper and it contains a cell association function for each tier that characterizes the average achievable downlink rate of a BS as well as the weighted power consumption of the BS. First, the cell association probability and the statistics of an associated BS in the \( k \)-th tier are derived under a very general universal cell association function, and then they are applied to find the association probability and the statistics of a \( k \)-th-tier BS for the EECA scheme. The coverage probability and average energy efficiency for EECA were also found. Finally, the load cell of a BS in any tier that characterizes the average number of users per cell is analyzed. From these derived and simulation results, we can learn that users tend to associate with a BS having a high bits-per-joule throughput so that doing traffic offloading via the EECA scheme can achieve a (much) higher coverage probability, average energy efficiency, and better cell load balancing than the nearest BS association (NBA) and maximum received power association (MRPA) schemes [5] when BSs are densely deployed in the network.

II. SYSTEM MODEL AND PRELIMINARIES

A. \( K \)-tier HetNet Modeling

Consider an infinitely large \( K \)-tier heterogeneous cellular network on the plane \( \mathbb{R}^2 \) where all base stations form \( K \) independent homogeneous Poisson point processes (PPPs). The BSs in the \( k \)-th tier are a marked PPP of intensity \( \lambda_k \).
and they can be specifically written as set $\Phi_k$ given by
\[ \Phi_k \triangleq \{ (B_{k_j}, H_{k_j}, P_k, \Psi_k) : B_{k_j} \in \mathbb{R}^2, \]
\[ H_{k_j}, \Psi_k, P_k \in \mathbb{R}_{++}, \}, \quad k \in K \triangleq \{1, 2, \ldots, K\}. \]  
(1)
where $B_{k_j}$ denotes the $j$th BS in the $k$th tier and its location, $H_{k_j}$ is the channel power gain between BS $B_{k_j}$ and the origin, $P_k$ stands for the power consumption of the BSs in the $k$th tier and it is given by
\[ P_k = P_k^{\text{in}} + \delta_k P_k \]  
(2)
in which $P_k^{\text{in}}$ is the power consumed by the hardware of a $k$th-tier BS, $P_k$ is the transmit power of a $k$th-tier BS and $\delta_k > 0$ is a scaling factor for $P_k$, and $\Psi_k : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is the (random) association reward function of BS $B_{k_j}$. Note that all $\Psi_k$’s are i.i.d. for all $j \in \mathbb{N}$ if they contain random variables and all channel power gains $H_{k_j}$’s are i.i.d. random variables with unit mean.

All users in the network also form an independent PPP of intensity $\lambda_u$. Without loss of generality, we assume there is a typical user $U_0$ located at the origin and our following analysis will be based on this typical user’s location. In this paper, we define the bits-per-joule energy efficiency of a $k$th-tier BS $B_{k_j}$, as follows
\[ \eta_k(||B_{k_j}||) \triangleq \frac{1}{P_k} \log_2 \left( 1 + P_k \mathbb{E} \left[ ||B_{k_j}||^{-\alpha} I_{k_j}^{-1} \right] \right), \]  
(3)
where $I_{k_j}$ is user $U_0$’s received interference while its serving BS is $B_{k_j}$ and $||X||$ is the Euclidean distance from node $X$ to the origin. The energy efficiency $\eta_k$ will be used in the EECA scheme for users to associate with their serving BSs in an energy-efficient fashion. Before addressing more about the EECA scheme, we need to first study the statistical properties of an associated BS in any tier under a very general universal cell association function, as shown in the following subsection.

B. Universal Cell Association and Its Statistical Results

Suppose every user in the network adopts universal cell association (UCA) to associate with its serving BS. Specifically, the UCA scheme for typical user $U_0$ can be written as
\[ B_{0}^u = \arg \sup_{B_{k_j} \in \bigcup_{k \in K} \Phi_k} \Psi_{k_j} (||B_{k_j}||) \]  
(4)
where $\Psi_{k_j} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is called the (random) UCA function for BSs in the $k$th tier. UCA is very general and can cover different kinds of cell association schemes. For example, users adopt (unbiased) nearest BS association (NBA) if $\Psi_{k_j} (||B_{k_j}||) = ||B_{k_j}||^{-\alpha}$, whereas they use instant maximum received power association if $\Psi_{k_j} (||B_{k_j}||) = P_k H_{k_j} ||B_{k_j}||^{-\alpha}$. In the following analysis, we will need the statistical properties of BS $B_{0}^u$, especially the distribution of its Euclidean distance to the origin which is shown in the following theorem.

**Theorem 1.** Suppose all users in the HetNet adopt the UCA scheme in (4) to do cell association. The UCA function $\Psi_{k_j}$ is an invertible and monotonic decreasing function and its inverse function is denoted by $\Psi_{k_j}^{-1}$. The $k$th-tier association probability that BS $B_{0}^u$ in (4) is from the $k$th tier is $\vartheta_k^u$ given by

\[
\vartheta_k^u = \mathbb{E} \left\{ \exp \left( -\pi \sum_{m \in K \setminus k} \lambda_m \mathbb{E} \left[ (\Psi_m^{-1} \circ \Psi_k(||B_m^u||))^2 | \Psi_k(||B_k^u||) \right] \right) \right\},
\]  
(5)
where $B_m^u \triangleq \arg \sup_{B_{k_j} \in \Phi_k} \Psi_{k_j}(B_{k_j})$ and $\Psi_m^{-1} \circ \Psi_k(\cdot)$ is the composition function of $\Psi_m^{-1}(\cdot)$ and $\Psi_k(\cdot)$. The cumulative density function (CDF) of $||B_k^u||$ is

\[
P(||B_k^u|| \leq x) = \frac{1}{\lambda_k} \mathbb{E}_{\Psi_k} \left\{ \exp \left( -\pi \lambda_k (\Psi_k^{-1}(\Psi_k(x)) x)^2 | \Psi_k^{-1}(\Psi_k(x)) \right) \right\},
\]  
(6)
where $\hat{\Psi}_k(\cdot)$ and $\Psi_k(\cdot)$ are i.i.d., and the CDF of the Euclidean distance of BS $B_0^u$ defined in (4) is

\[
P(||B_0^u|| \leq x) = 1 - \sum_{k \in K} \mathbb{E}_{\Psi_k} \left\{ \exp \left( -\sum_{m \in K \setminus k} \pi \lambda_m \mathbb{E} \left[(\Psi_m^{-1} \circ \Psi_k(x))^2 | \Psi_k(x) \right] \right) \right\}.
\]  
(7)

**Proof:** See Appendix A.

The results in Theorem 1 are suitable for any cell association schemes due to the generality of UCA. They can be largely simplified if the cell association reward function for every BS is deterministic and identical for the same tier, as shown in the following corollary.

**Corollary 1.** If the UCA function for every BS is deterministic, $\vartheta_k^u$ in (5), the CDF of $||B_k^u||$ in (6), and the CDF of $||B_0^u||$ in (7) can reduce to the following results, respectively:

\[
\vartheta_k^u = \mathbb{E} \left[ e^{-\pi \sum_{m \in K \setminus k} \lambda_m |\Psi_m^{-1} \circ \Psi_k(||B_m^u||)|^2} \right],
\]  
(8)
\[
P(||B_k^u|| \leq x) = 1 - \exp \left( -\pi \lambda_k x^2 \right),
\]  
(9)
\[
P(||B_0^u|| \leq x) = 1 - \sum_{k \in K} e^{-\pi \sum_{m \in K \setminus k} \lambda_m |\Psi_m^{-1} \circ \Psi_k(x)|^2}.
\]  
(10)

**Proof:** Since all the cell association reward functions are deterministic, $\mathbb{E} \left[ |\Psi_m^{-1} \circ \Psi_k(||B_m^u||)|^2 | \Psi_k(||B_k^u||) \right] = |\Psi_m^{-1} \circ \Psi_k(||B_m^u||)|^2$ for $m \neq k$ and thus (5) reduces to (8), Eqs. (6) and (7) respectively become (9) and (10) because of $\Psi(\cdot) = \hat{\Psi}(\cdot)$ and $\mathbb{E} \left[ (\Psi_m^{-1} \circ \Psi_k(x))^2 \right] = x^2$.

The $k$th-tier cell association probability in (8) is more general than some similar results in the literature, such as [5], [9]. The following lemma shows this.

**Lemma 1.** The probability that $B_0^u$ (NBA) or $B_0^m$ (MRPA) is from $\Phi_k$ can be expressed as:

\[
\vartheta_k^u = \frac{\lambda_k}{\sum_{m=1}^{K} \lambda_m}, \quad \vartheta_k^m = \frac{\lambda_k P_k^{2/\alpha}}{\sum_{m=1}^{K} \lambda_m P_m^{2/\alpha}},
\]  
(11)

**Proof.** The proof follows from the application of (8) and (9) on NBA and MRPA scheme. See also [5].
Additionally, (9) is exactly equal to the CDF of the Euclidean distance between the nearest BS in the kth tier and the origin. This is because the UCA function is deterministic and monotonic decreasing along the distance. Furthermore, the CDFs of $||B_0^u||$ in (7) and (10) are important since they can be used to calculate many network performance metrics under any cell association schemes, such as coverage probability, average user throughput, average transmit power consumption, etc. In the next section, we will focus on the study of energy-efficient cell association and the results in Theorem 1 will significantly help simplify the derivation of the CDF of the distance from the associated BS to the origin.

III. ENERGY-EFFICIENT CELL ASSOCIATION (EECA)

The UCA results described in Section II-B can be transformed to energy-efficiency-based cell association (EECA) if the UCA function in (4) is designed based on the energy-efficiency in (3), i.e., $\Psi_{k_j}$ is a certain function of $\eta_{k_j}$. Specifically, we assume all users adopt the following EECA scheme

$$B_0^u = \arg \sup_{B_k} \frac{R_k(||B_k||)}{\Xi_k},$$

where $\Xi_k = \omega_k P_k$ is a weighted value of $P_k$ and $\omega_k$ is a positive constant for controlling the traffic offloaded to the BSs in the kth tier. For simplicity, the downlink rate function for BS $B_k$ is designated as the upper bound on the average spectrum efficiency provided by the BS, i.e.,

$$R_k(||B_k||) \triangleq \log_2 \left(1 + \frac{P_k H_{k_j}}{I_{k_j} ||B_k||^{-\alpha}}\right),$$

where $I_{k_j} = \sum_{m=K_j} P_m H_{m_i} ||B_m - B_j||^{-\alpha}$ is the interference power observed by BS $B_{k_j}$. Note that all $I_{k_j}$'s and $P_k H_{k_j}$'s for all $j$ and $k$ are independent owing to the Slivnyak theorem [10], [11].

As a result of the independence within the expectation operator in (13), the closed-form result of $R_k(||B_k||)$ can be found and it is given in the following theorem.

**Theorem 2.** The downlink rate function for a given BS $B_{k_j}$ in (13) can be shown as

$$R_k(||B_k||) = \log_2 \left(1 + c_k ||B_k||^{-\alpha}\right),$$

where $c_k = P_k \Gamma \left(1 + \frac{1}{\alpha}\right) \left(\frac{\pi \tau_\alpha}{\alpha} \sum_{m=1}^K \lambda_m \frac{P_m^2}{\bar{H}_m^2}\right)^{-\frac{1}{\alpha}},$ $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ is the Gamma function, and $\Gamma(x) \triangleq \Gamma \left(1 + \frac{1}{\alpha}\right) \frac{\bar{H}_k^2}{\alpha}.$

**Proof:** See Appendix B.

Since all data in (14) are available for all BSs in the network, BSs can easily calculate their energy-efficiency cell association function and send them to users. Note that (14) is valid for any distribution of channel power gain $H$ and we can use it to find the statistics of the distance of BS $B_0^u$ to the origin, as shown in the following subsection.

A. The Statistics of $||B_0^u||$

Since $R_k(||B_k||)$ in (14) is a deterministic, invertible and monotonic decreasing function of $||B_k||$, we can use Corollary 1 and Theorem 2 to characterize the statistics of distance $||B_0^u||$, which is summarized in the following corollary.

**Corollary 2.** Suppose each user adopts the EECA scheme in (12) to associate with its serving BS in the K-tier HetNet and the downlink rate function $R_k$ of the typical user is given in (14). Then the probability of $B_0^u$ in the kth tier is

$$P[||B_0^u|| \leq x] = 1 - \sum_{k \in K} \int_0^\infty e^{- \sum_{m=1}^K \lambda_m \frac{P_m^2}{\bar{H}_m^2}} \sum_{u=0}^{\alpha-1} \frac{\sum_{m=1}^K \lambda_m \frac{P_m^2}{\bar{H}_m^2}}{\lambda_k \frac{P_k^2}{\bar{H}_k^2}} x^u du$$

**Proof:** The result in (15) can be directly obtained from (8) by using (9) and plugging (12) into (10) gives (16).

By assuming $c_k (\pi \lambda_k / y)^{\frac{1}{\alpha}} \ll 1$, $\vartheta_k$ in (15) can be approximately calculated as

$$\vartheta_k \approx \frac{\lambda_k (P_k / \Xi_k)^{\frac{1}{\alpha}}}{\sum_{m=1}^K \lambda_m (P_m / \Xi_m)^{\frac{1}{\alpha}}},$$

which indicates that users prefer to connect to the BSs in the tier with a high ratio of transmit power to total power consumption. Corollary 2 is very useful for deriving some performance metrics of EECA. In particular, we discuss the following two typical metrics – coverage probability and average energy efficiency. We first discuss the coverage probability of a user.

1) Coverage Probability: For any $\theta > 0$, the coverage probability of a user is typically defined as $P[SIR_0^u \geq \theta]$ in which $SIR_0^u$ is the signal-to-interference ratio (SIR) of user $U_0$ and it is defined as

$$SIR_0^u \triangleq \sum_{k \in K} \frac{P_k H_k ||B_0^u||^{-\alpha} I_k (B_0^u \in \Phi_k)}{P_k},$$

where $I_k = \sum_{m=1}^K \sum_{k \in K_j} P_m H_{m_i} ||B_m - B_j||^{-\alpha}$ is the interference power received by user $U_0$. By assuming all channel power gains are i.i.d. exponential random variables with unit mean and variance, we thus obtain

$$P[SIR_0^u \geq \theta] = \sum_{k \in K} \mathbb{E} \left[ \exp \left(-\theta \frac{P_k H_k ||B_0^u||^{-\alpha} I_k}{P_k} \right) \right] \vartheta_k,$$

where $I_k = \sum_{m=1}^K \sum_{k \in K_j} P_m H_{m_i} ||B_m - B_j||^{-\alpha}$ and $\vartheta_k$ is given in (15). Also, note that (17) is an MRPA scheme in disguise where it is biased by $\Xi_k$ for some $k \in K$. This
energy efficiency of the EECA scheme is defined as

$$\mathbb{P}[\| B_0^e \| \leq x \mid B_0^e \in \Phi_k] \approx \frac{1 - e^{-\pi x^2 \sum_{m \in k} \lambda_m \left( \frac{P_m / \Xi_m}{P_k / \Xi_k} \right)^{\frac{2}{\alpha}}}}{c_k \Xi_k^{\frac{2}{\alpha}}}.$$  (20)

Then, applying the previous results in [12], [13], and probability density function (pdf) of (20) yields

$$\mathbb{E} \left[ \exp \left( -\frac{\| B_0^e \|^\alpha}{P_k} I_k^e \right) \mid B_0^e \in \Phi_k \right] \approx \left[ 1 + \frac{s_k}{\mathcal{A}_k} \right]^{-1} \sum_{m \in k} \lambda_m P_m \frac{2}{\alpha} \epsilon (s_k P_m, z_m),$$

$$\sum_{m \in k} \lambda_m P_m \frac{2}{\alpha} \epsilon (s_k P_m, z_m)$$

where

$$s_k = \frac{\theta \| B_0^e \|^\alpha}{P_k}, \mathcal{A}_k = \sum_{m \in k} \lambda_m \left( \frac{P_m / \Xi_m}{P_k / \Xi_k} \right)^{\frac{2}{\alpha}},$$

and

$$\epsilon (a, b) = \int_{b a^{-2/\alpha}}^1 \frac{du}{e^{u a^{2/\alpha}}},$$

Distance of nearest interfering node from each tier is approximated by

$$z_k \approx \| B_0^e \| \sqrt{P_0 / P_k}.\$$

Note that the coverage probability of the EECA scheme in (19) can reduce to the coverage probability of some other cell association schemes. For example, the coverage probability of MRPA can be readily acquired by replacing $\vartheta_k^e$ in (19) with corresponding probability in Lemma 1, averaging over distance of $B_0^e$, and setting $\Xi_1 = \cdots = \Xi_K = 1$.

2) Average energy efficiency: Based on (3), the average energy efficiency of the EECA scheme is defined as

$$\eta^e \triangleq \sum_{k=1}^K \vartheta_k^e \log_2 \left( 1 + P_k \mathbb{E} \left[ \| B_0^e \|^\alpha / I_k^e \right] \right),$$

where $I_k^e$ is the received interference of the typical user served by BS $B_0^e$. Since $\| B_0^e \|^\alpha$ and $I_k^e$ are correlated, the closed-form of $\eta^e$ cannot be derived. Similarly, the average energy efficiencies of the NBA and MRPA schemes can be defined as shown in the following:

$$\eta^n \triangleq \sum_{k=1}^K \lambda_k / P_k \sum_{m=1}^{K} \lambda_m \log_2 \left( 1 + P_k \mathbb{E} \left[ \| B_0^e \|^\alpha / I_k^e \right] \right),$$

$$\eta^m \triangleq \sum_{k=1}^K \lambda_k P_k^2 \sum_{m=1}^{K} \lambda_m P_m \log_2 \left( 1 + P_k \mathbb{E} \left[ \| B_0^e \|^\alpha / I_k^e \right] \right).$$

More discussions about the coverage probabilities and the average energy efficiencies of different cell association schemes will be elaborated via the numerical results in Section IV.

### B. Cell Load Analysis

In the previous section, we studied the statistics of an associated BS for the EECA scheme and use it to find the two performance metrics, coverage probability and average energy efficiency. Another very important metric of evaluating the performance of EECA that interests us is the cell load, which is defined as the average number of users associated in a cell. To characterize the cell load of a BS in the $k$th-tier, we need to find the cell coverage of the BS in which each user using EECA necessarily connects to this BS. The cell coverage of BS $B_0^e$ is defined as

$$C_0^e \triangleq \left\{ U \in \mathbb{R}^2 \mid \frac{\mathcal{R}_k(\| B_0^e - U \|)}{\mathbb{E} \left[ \| B_0^e - U \| \right]} \geq \frac{\Xi_k}{\Xi_m} - k, m \in K, \right\} \quad \forall B_{m}, \in \bigcup_{m \in K} \Phi_m \setminus B_0^e,$$

i.e., any user using EECA will associate with BS $B_0^e$ if it is in $C_0^e$. The Lebesgue measure of $C_0^e$, denoted by $|C_0^e|$, is a random variable and its moment is given in the following theorem.

Theorem 3. For any $\mu > 1$, the $\mu$-moment of the Lebesgue measure of $C_0^e$ in (22) can be shown as

$$\mathbb{E}[|C_0^e|^\mu] = (1 + \mu) \sum_{k=1}^K \frac{\vartheta_k^e}{\lambda_k^\mu},$$

$$\approx (1 + \mu) \sum_{k=1}^K \frac{\lambda_k \sum_{m=1}^K \lambda_m (P_m / \Xi_m)^{\frac{2}{\alpha}}}{\sum_{m=1}^K \lambda_m (P_m / \Xi_m)^{\frac{2}{\alpha}}}.$$

Proof: According to the result in (16) and Slivnyak’s theorem, for any $U \in \mathbb{R}^2$, we can infer the following result:

$$\mathbb{P} \left[ (\pi\| U - B_{k,j} \|^2)^\mu \geq y \right] = \sum_{k=1}^K \exp \left( -\lambda_k y^{\frac{1}{1+\mu}} \right) \vartheta_k^e.$$}

Thus, $\mathbb{E}[|C_0^e|^\mu]$ can be calculated by

$$\mathbb{E}[|C_0^e|^\mu] = \int_0^\infty \mathbb{P} \left[ (\pi\| U - B_{k,j} \|^2)^\mu \geq y \right] dy$$

Carrying out this integral yields (23) and the approximated result is obtained by substituting (17) into (23).

According to Theorem 3, we can infer the cell load $\ell_k^e$ of a BS in the $k$th tier, i.e.,

$$\ell_k^e \triangleq \mathbb{E}[|C_0^e| \mathbb{E}_k | B_0^e \in \Phi_k],$$

as follows

$$\ell_k^e = \frac{\lambda_k \vartheta_k^e}{\sum_{m=1}^K \lambda_m (P_m / \Xi_m)^{\frac{2}{\alpha}}}.$$

because $\lambda_k \vartheta_k^e$ can be viewed as the intensity of the user associated with the $k$th-tier BSs and the average cell area in that tier is $1/\lambda_k$. Obviously, if we adjust all $\omega_k$’s such that $P_0^e = P_2^e = \cdots = P_K^e$, all cells almost have the same load, i.e., the network is nearly load-balanced. Similarly, $\ell_k^e$ in (25) can reduce to the cell load of some other cell association schemes by changing $\vartheta_k^e$. For the case of maximum received power association, the cell load of a $k$th-tier BS is

$$\ell_k^e = \frac{\lambda_k P_k^2}{\sum_{m=1}^K \lambda_m P_m^2},$$

and this association scheme could have a fairly unbalanced load if the transmit powers for different tiers have big differences. Thus, MRPA needs to have biased weights to improve load balancing. The feature of EECA from the viewpoint of cell load is that the load of a cell will become heavier if the cell has a higher energy efficiency. Accordingly, users tend to
TABLE I
NETWORK PARAMETERS FOR SIMULATION [14]

<table>
<thead>
<tr>
<th></th>
<th>Macrocell</th>
<th>Picocell</th>
<th>Femtocell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmit power (W), $P_k$</td>
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<td>2</td>
<td>0.05</td>
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<tr>
<td>Hardware Power (W), $P_{on}^k$</td>
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<td>6.8</td>
<td>4.8</td>
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<tr>
<td>Weight $\omega_k$ of EECA</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>2.66</td>
<td>4.0</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Fig. 1. Numerical results of the coverage Probabilities. The threshold $\theta$ of the SIR is 0.5.

associate with a BS with low energy cost, which is usually a small cell BS, such as a picocell or femtocell. Although other cell association schemes can also make macro BSs offload traffic to small cell BSs, this traffic offloading process in general cannot attain a high energy efficiency. The numerical results in the following section will illustrate this point.

IV. NUMERICAL RESULTS

In this section, we present some numerical results of coverage probability, average energy efficiency and cell load for a three-tier HetNet where the first tier consists of macro BSs whereas picocells and femtocell form the second and third tier, respectively. The network parameters for simulation are listed in the Table I. The intensities of the macro BSs and pico BSs are respectively $\lambda_1 = 1$ BSs/Km$^2$ and $\lambda_2 = 50$ BSs/Km$^2$. We vary $\lambda_3$ over $\lambda_1$ for analytical purposes. The path loss exponent for all tiers is $\alpha = 4$. The numerical results of the coverage probability, average energy efficiency and cell load for the EECA, MRPA and NBA schemes are presented in Figs. 1, 2 and 3, respectively.

The coverage probability of MRPA in Fig. 1 is much better than NBA but only by a small margin over the EECA scheme. Under an interference-limited network with identical $\alpha$, the MRPA scheme gives a constant coverage probability regardless of BS intensity variations. The EECA scheme performs reasonably well with coverage probability almost near the MRPA’s. The results indicate that the strongest received power factor should be considered in the association function in a dense HetNet. Fig. 2 shows the average energy efficiencies and EECA is superior to the other two schemes, as expected. Specifically, the minor degradation in coverage probability for the EECA is compensated by best overall energy efficiency among the three schemes. Finally, the numerical results of the cell loads are shown in Fig. 3. We can observe that the cell loads of EECA and NBA are much more balanced than MRPA since MRPA essentially tends to make users associate with macro BSs which have strong transmit powers.

V. CONCLUSION

An energy-efficient cell association scheme is proposed based on a designated bits-per-joule-based energy-efficient cell association function. The general statistical results of an associated BS in any tier are first investigated for a universal cell association function. They can be easily transformed to the case of all other cell association schemes. With the statistical results of an associated BS in the $k$th-tier for EECA, the coverage probability, average energy efficiency and cell load can all be calculated. We show that EECA can outperform the MRPA in terms of coverage probability, average energy efficiency and cell load balancing and it is also better than NBA in terms of energy efficiency and cell load balancing. Our future work will study the energy-efficient throughput per
cell or per user and how biased EECA affects the performance of traffic offloading.

APPENDIX

A. Proof of Theorem 1

First define $B_k^u$ as the base station having the maximum cell association reward in the $k$th tier, i.e. $B_k^u \triangleq \arg \max_{B_j \in \Phi_k} \Psi_k(\|B_j^u\|)$. According to the definition of $\hat{\Psi}_k$, we have the following identity

$$P[\|B_k^u\| \leq x] = \mathbb{E}_{\hat{\Psi}_k} \left\{ P \left[ \sup_{B_j \in \Phi_k} \Psi_k(\|B_j^u\|) \geq \hat{\Psi}_k(x) \right] \right\}. \quad (27)$$

Let $\hat{\Psi}_k(\|B_k^u\|) \overset{d}\triangleq \sup_{B_j \in \Phi_k} \Psi_k(\|B_j^u\|)$ where $\overset{d}\triangleq$ stands for equivalence in distribution. Thus, we have

$$P \left[ \hat{\Psi}_k(\|B_k^u\|) \geq y \right] = P \left[ \sup_{B_j \in \Phi_k} \Psi_k(\|B_j^u\|) \geq y \right] = 1 - \prod_{B_j \in \Phi_k} P[\Psi_k(\|B_j^u\|) \leq y] \quad (\star)$$

$$= 1 - \exp \left( -\lambda_k \int_{\mathbb{R}^2} (1 - \mathbb{P}[\Psi_k(\|B_j^u\|) \leq y]) \, dB_k \right)$$

$$= 1 - \exp \left( -\pi \lambda_k \mathbb{E}[\|\hat{\Psi}_k^{-1}(y)\|^2] \right), \quad (28)$$

where $(\star)$ follows from the probability generating functional of a homogeneous PPP [10] [12].

Substituting (28) into (27) leads to $P[\|B_k^u\| \leq x]$ in (6). Also, from (28) we know the following

$$P \left[ \max_{m \in K} \Psi_m(\|B_m^u\|) \leq x \right] = \prod_{m \in K} P[\Psi_m(\|B_m^u\|) \leq x] = \exp \left( -\pi \sum_{m \in K} \lambda_m \mathbb{E}[\|\hat{\Psi}_m^{-1}(x)\|^2] \right).$$

Then the probability that $B_0^u$ is from the $k$th tier can be expressed as

$$\vartheta_k^u \triangleq P[B_0^u \in \Phi_k] = \prod_{m \in K^c_k} P\left[ \max_{m \in K^c_k} \Psi_m(\|B_m^u\|) \leq \hat{\Psi}_k(\|B_0^u\|) \right]$$

$$= \prod_{m \in K^c_k} \mathbb{E}\left\{ \mathbb{P}\left[ \max_{m \in K^c_k} \Psi_m(\|B_m^u\|) \leq \hat{\Psi}_k(\|B_0^u\|) \right] \right\},$$

which gives the result in (5) by plugging (27) into it. The conditional CDF of $\|B_0^u\|$ given that $B_0^u \in \Phi_k$ is written as

$$P[\|B_0^u\| \leq x | B_0^u \in \Phi_k] = \frac{P[\|B_0^u\| \leq x, B_0^u \in \Phi_k]}{P[B_0^u \in \Phi_k]}, \quad (29)$$

where

$$P[\|B_0^u\| \leq x, B_0^u \in \Xi_k] = \prod_{m \in K} \mathbb{P}\left[ \max_{m \in K^c_k} \Psi_m(\|B_m^u\|) \leq \hat{\Psi}_k(x) \right]$$

$$= 1 - \mathbb{E}_{\hat{\Psi}_k} \left[ \exp \left( -\pi \sum_{m \in K} \mathbb{E}\left[ \left( \Psi_m^{-1} \circ \hat{\Psi}_k(x) \right)^2 \right] \right) \right].$$

The law of total probability gives

$$P[\|B_0^u\| \leq x] = \sum_{k \in K} P[\|B_0^u\| \leq x | B_0^u \in \Phi_k] \vartheta_k^u. \quad (30)$$

Substituting (29) into (30) yields (7).

B. Proof of Theorem 2

First of all, we notice that

$$\mathbb{E} \left[ P_k H_{k_j} I_{k_j}^{-1} \|B_k^u\|^{-\alpha} | B_j \right] = P_k \mathbb{E} \left[ I_{k_j}^{-1} \right] \|B_k^u\|^{-\alpha}$$

because $H_{k_j}$ and $\|B_k^u\|$ are independent of $I_{k_j}$ and $\mathbb{E}[H] = 1$.

Then we know $\mathbb{E} \left[ I_{k_j}^{-1} \right]$ can be rewritten as

$$\mathbb{E} \left[ I_{k_j}^{-1} \right] = \int_0^\infty e^{-s I_{k_j}} \, ds = \int_0^\infty \mathbb{E} \left[ e^{-s I_{k_j}} \right] \, ds$$

$$= \int_0^\infty \exp \left( -\pi \tau_{\alpha} \left( \sum_{m=1}^K \lambda_m P_m^2 \right) s^2 \right) \, ds$$

$$= \int_0^\infty t^{\frac{1}{2}} \exp \left( -\pi \tau_{\alpha} \left( \sum_{m=1}^K \lambda_m P_m^2 \right) t^2 \right) dt$$

$$= \Gamma \left( 1 + \frac{\alpha}{2} \right) \left( \pi \tau_{\alpha} \sum_{m=1}^K \lambda_m P_m^2 \right)^{-\frac{\alpha}{2}},$$

where $(\star)$ follows from the result in [12]. Thus, we have

$$\mathbb{E} \left[ P_k H_{k_j} I_{k_j}^{-1} \|B_k^u\|^{-\alpha} \right] = c_k \|B_k^u\|^{-\alpha}$$

and substituting this result into (13) leads to (14).

REFERENCES