A Generalized Analytical Framework for Coverage Evaluation in mmWave Heterogeneous Cellular Networks in Urban Areas

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Abstract—In this paper, we introduce a generalized analytical framework to evaluate the coverage probability in an mmWave heterogeneous cellular network (HetNet) consisting of UHF macrocell BSs and mmWave small cell BSs deployed in an urban area. A generalized user association scheme that can cover a few user association schemes is formulated and applied in the mmWave HetNet. Two important statistical properties of the associated BS under this scheme are derived for the general line-of-sight (LOS) and non line-of-sight (NLOS) pathloss channel models and they can be largely simplified if the power-law channel model is applied. Using the two statistical properties, the coverage probability of a downlink multiple-input-single-output (MISO) channel is found in a compact form, which reveals that the coverage probability in the mmWave HetNet can be significantly improved if compared with the cellular network consisting of a single tier of mmWave small cell BSs. Numerical results not only validate the correctness of the coverage probability derived but also indicate that the mmWave HetNet has the potential to achieve a high coverage probability in an urban area.

I. INTRODUCTION

Due to the proliferation of wireless smart handsets and devices, cellular data traffic is expected to tremendously grow to satisfy customers’ huge and different throughput demands in different networking services. To make cellular networks jump over the high throughput hurdle due to limited licensed spectrum (band), densely deploying mmWave small cells is a promising approach to alleviating the spectrum crunch problem in the near future. However, mmWave signals suffer high path and penetration losses so that the transmission performance of mmWave BSs in an urban area where there are a lot of blockages that severely impede the propagations of mmWave signals. Thus, how to deploy mmWave small cells in a dense blockage environment so that most users can connect to mmWave small cells (BSs) to enjoy extremely high throughput due to a large available bandwidth in the mmWave band is a challenging problem.

In the future architecture of a cellular network, UHF BSs and mmWave BSs are foreseen to coexist in a heterogeneous cellular network in order to get rid of the spectrum crunch problem and achieve the goal of high coverage and throughput. Thus, how to evaluate the transmission performance (such as coverage and throughput) in this kind of HetNets is a paramount problem that needs to be thoroughly studied. Almost all the prior works focus on the coverage analysis in a single-tier mmWave cellular network. For example, reference [1] studied the coverage and rate problems in a single-tier mmWave cellular network and only the approximated analytical results of the coverage probability and rate are obtained. In [2], the rate problem was studied in a single-tier mmWave cellular network with a limited self-backhaul resource. Although a recent work in [3] indeed studied the coverage problem in a mmWave heterogeneous cellular, it only focused on the uplink and downlink decoupling scenario and did not investigate how different user association schemes affects the coverage performance.

In this work, we aim to exhaustively and generally study the coverage problem in an mmWave heterogeneous cellular consisting of multiple tiers of the UHF macrocell and small cell BSs and a single tier of mmWave small cell BSs. We develop a generalized analytical framework based on stochastic geometry to first study some statistically properties of the generalized user association scheme that characterizes the LOS and NLOS channel models and then use these properties to derive the coverage probability of a user with a downlink MISO channel. The derived coverage probability can give us some insights into how the coverage probability is contributed by each tier of BSs. Most importantly, it shows that a very high coverage probability can be achieved in this mmWave HetNet even the blockages may weaken the coverage probability contributed by the mmWave small cell BSs. Simulation results reveal that our analytical findings are fairly accurate and densely deploying too many mmWave small cell BSs in an urban area may not effectively increase the coverage probability since the high penetration loss and interference of the mmWave BSs limits the increase in the signal-to-interference plus noise power ratio (SINR) at users.

II. NETWORK MODEL

In this paper, we consider an $M$-tier planar heterogeneous cellular network (HetNet) in which all base stations (BSs) in any particular tier that have the same type and performance form an independent Poisson point process (PPP) with certain intensity. To characterize the situation that traditional UHF/microwave BSs and mmWave small cell BSs coexist in
this HetNet, we assume the first \( M - 1 \) tiers consist of the UHF macrocell and small cell BSs whereas the \( M \)th tier consists of the mmWave small cell BSs. For the BSs in the \( n \)th tier, they can be written as a homogeneous PPP of intensity \( \lambda_m \) given by
\[
\Phi_m \triangleq \left\{ X_{m,i} \in \mathbb{R}^2 : m \in \mathcal{M} \triangleq \{1, \ldots, M\}, i \in \mathbb{N}_+ \right\},
\]
where \( X_{m,i} \) denotes BS \( i \) in the \( n \)th tier and its location.

Without loss of generality, assume there is a typical user located at the origin and our following location-dependent expressions and analyses are based on this typical user\(^1\). Also, we consider the mmWave HetNet is in an urban area where the centers of all blockages, such as buildings, towers, houses, obstacles, etc., are also assumed to form an independent PPP of intensity \( \beta \) for analytical tractability and we use a Boolean scheme of rectangles to model blockages in an urban area. With considering the blockages effects on the transmission channel between a BS and its serving user, the channel can be a line-of-sight (LOS) or non-line-of-sight (NLOS) channel depending on whether the channel is visually blocked between the BS and its user. LOS and NLOS channels induced by urban blockages have a very distinct impact on the transmitted signal powers, especially the mmWave signal powers. In the following subsection, we will consider a generalized user association scheme that characterizes the user association signal (usually called primary synchronization signal in an LTE system) periodically broadcasted by BSs.

A. Generalized user association scheme

In this HetNet, users associate with their serving BS by using the following generalized user scheme that is based on the location of the typical user:
\[
X_* = \arg \Psi_*(||X_*||) = \arg \sup_{m,i \in \Phi} \Psi_{m,i}(||X_{m,i}||),
\]
where \( X_* \in \Phi \triangleq \bigcup_{m=1}^{M} \Phi_m \) denotes the BS associated with the typical user, \( ||X_i - X_j|| \) denotes the Euclidean distance between BSs \( X_i \) and \( X_j \) for \( i \neq j \), \( \Psi_m : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is called the user association function of BS \( X_{m,i} \) and \( \Psi_* \in \{\Psi_{m,i}, m \in \mathcal{M}, i \in \mathbb{N}_+\} \) is the user association function of BS \( X_* \). Since all BSs are in a urban environment, whether their channels to their users are LOS or NLOS is seriously affected by the blockages (especially for the mmWave BSs whose LOS and NLOS channels behave very differently,) so that we propose the following user association function \( \Psi_{m,i} \) for BS \( X_{m,i} \) that characterizes the LOS and NLOS statuses of the downlink channel of BS \( X_{m,i} \):
\[
\Psi_{m,i}(||X_{m,i}||) \triangleq \ell(||X_{m,i}||) \Psi_{m,i}(||X_{m,i}||)
+ \left(1 - \ell(||X_{m,i}||)\right) \Psi_{m,i}(||X_{m,i}||),
\]
where \( \ell(r) \in \{0, 1\} \) denotes a Bernoulli RV that is one if there are no blockages within distance \( r \) and zero otherwise, and

The distribution of \( \Psi_*(||X_*||) \) is important for our subsequent analyses and it is shown in the following theorem.

**Theorem 1.** Suppose the user association scheme in (2) is adopted by users. If \( \Psi_{m,i}(-) \) and \( \Psi_{m,i}(\cdot) \) both are bijection, the cumulative density function (CDF) of \( \Psi_*(||X_*||) \) is
\[
F_{\Psi_*(||X_*||)}(x) = \exp\left(-\pi \sum_{m=1}^{M} \lambda_m \mathcal{A}_m(x)\right),
\]
where \( \mathcal{A}_m(x) \) is defined as
\[
\mathcal{A}_m(x) \triangleq 2E \left[\int_{0}^{x/2} te^{-nt^2} dt + E \left(\tilde{\Psi}_{m}^{-1}(x)\right)^2\right],
\]
where \( g^{-1}(\cdot) \) denotes the inverse of real-valued function \( g(\cdot) \). The tier-m association probability that a user associates with a tier-m BS is
\[
\phi_m = 2\pi \lambda_m E_{\Psi_m} \left\{\int_{0}^{\infty} e^{-\pi \sum_{k=1}^{M} \lambda_k A_k \circ \psi_m^k(x) dx}\right\},
\]
where composition function \( A_k \circ \psi_m^k(x) \) for given \( \Psi_m^k(x) \) is defined as
\[
\mathcal{A}_k \circ \psi_m^k(x) = E \left[\int_{0}^{x/2} \frac{2t}{e^{nt^2}} dt + \left(\tilde{\Psi}_{k}^{-1} \circ \tilde{\Psi}_{m}^{-1}(x)\right)^2\right]
\]
in which \( g_1 \circ g_2(x) = g_1(g_2(x)) \) is the composition of functions \( g_1(\cdot) \) and \( g_2(\cdot) \), and functions \( g_m(\cdot) \) and \( \mathcal{A}_m(\cdot) \) are i.i.d. if they are random.

**Proof:** See Appendix A.

The CDFs regarding to BS \( X_* \) shown in Theorem 1 are so general that they not only work for any invertible user associate functions but also reflect the impacts of LOS and NLOS channel effects due to blockages. For example, \( \mathcal{A}_m(x) \) in (5) can be explicitly found if \( \Psi_m(x) \) and \( \Psi_m(x) \) are designed as an invertible power-law pathloss function of \( x \) as shown in the following subsection. Furthermore, we can realize
\[
\mathbb{E} \left(\tilde{\Psi}_{m}^{-1}(x)^2\right) \leq \mathcal{A}_m(x) \leq \mathbb{E} \left(\tilde{\Psi}_{m}^{-1}(x)^2\right)
\]
because
\[
\lim_{\beta \to \infty} \mathcal{A}_m(x) = E \left(\tilde{\Psi}_{m}^{-1}(x)^2\right) \quad \text{(i.e., an infinitely large}
\]
large blockage intensity makes all channels become LOS) and
\[
\lim_{\beta \to 0} \mathcal{A}_m(x) = E \left(\tilde{\Psi}_{m}^{-1}(x)^2\right) \quad \text{(i.e., all channels are LOS since no blockages). Since in general}
\]
\[
\mathbb{E} \left(\tilde{\Psi}_{m}^{-1}(x)^2\right) \leq E \left(\tilde{\Psi}_{m}^{-1}(x)^2\right), \text{modeling all channels as NLOS, which is the}
\]
most popular modeling assumption made in the prior works, may significantly impact the accuracy of the analytical results.

\(^1\)According to the Slivnyak theorem, the statistical properties observed by the typical user located at the origin are the same as those observed by users in any other locations in the network [4].
B. Pathloss and channel models for UHF and mmWave BSs

Pathloss Models. The signals of all BSs undergo pathloss before they arrive serving user. In this paper, we consider the following pathloss $L_{m,i}(\cdot)$ between BS $X_{m,i}$ and the typical user:

$$L_{m,i}([X_{m,i}]) = \nu_{\mu}([X_{m,i}]^{\alpha_{m,i}} \mathbb{1}(X_{m,i} \notin \Phi_M) + \nu_{\psi}([X_{m,i}]^{\alpha_{m,i}} \mathbb{1}(X_{m,i} \in \Phi_M),$$

where $[X_{m,i}]$ denotes the distance between BS $X_{m,i}$ and the typical user, $\mathbb{1}(E)$ is the indicator function that is equal to one if event $E$ is true and zero otherwise, $\nu_{\mu}$ and $\nu_{\psi}$ denote the intercepts of the UHF signals and the mmWave signals, respectively, and $\alpha_{m,i}$ is called the pathloss exponent that characterizes the LOS pathloss exponent $\alpha$ and the NLOS pathloss exponent $\bar{\alpha}$ of BS $X_{m,i}$, and it is defined as

$$\alpha_{m,i} \triangleq \ell([X_{m,i}])\alpha + [1 - \ell([X_{m,i}])]\bar{\alpha}. \tag{9}$$

Note that we assume $\alpha < \bar{\alpha}$ since LOS channels usually should have less pathloss than NLOS channels. According to [7], we know $\mathbb{E}[\ell(r)] = 1 = e^{-\eta r^2}$ where $\eta$ is a geometric parameter regarding to the mean perimeter of blockages.

The results in Theorem 1 regarding the general user association can be simplified to make themselves much implementable based on the pathloss model in (8). Namely, we can consider the user association function in (3) as a power-law based function pertaining to the pathloss of the BSs so that we have

$$\Psi_{m,i}([X_{m,i}]) = \psi_{m,i}[X_{m,i}]^{-\alpha_{m,i}}, \tag{10}$$

where $\psi_{m,i} \triangleq \ell([X_{m,i}])\overline{\psi}_{m,i} + [1 - \ell([X_{m,i}])]\hat{\psi}_{m,i}$ and $\alpha_{m,i}$ is already given in (9). Parameter $\overline{\psi}_{m,i}$ ($\hat{\psi}_{m,i}$) can be viewed as the (random) pathloss bias when BS $X_{m,i}$ has an LOS (NLOS) channel. Based on the user association function in (10), we simplify the results in Theorem 1 in the following corollary.

**Corollary 1.** If the user association functions of BS $X_{m,i}$ in (10) is adopted, the results in (5) and (7) reduce to the following:

$$A_m(x) = \mathbb{E} \left[ \int_{(\overline{x}_{m,i})^{1/\alpha}}^{x} 2te^{-\eta x^2} dt \right] + \mathbb{E} \left[ \left( \frac{\overline{\psi}_{m,i}}{x} \right)^{\frac{2}{\alpha}} \right], \tag{11}$$

$$A_k \circ \Psi_{m,i}(x) = x^2 \mathbb{E} \left[ \int_{x_k \overline{\psi}_{k,m}}^{x_k} \frac{2t}{e^{x_k^2 t}} dt \left| \psi_{m,i}(x) \right| \right] + \mathbb{E} \left[ \frac{2}{x_k \overline{\psi}_{k,m}} \left| \overline{\psi}_{m,i} \right| \right], \tag{12}$$

where $\overline{\psi}_{k,m} \triangleq \overline{\overline{\psi}}_{k,m}$ and $\hat{\psi}_{m,i} \triangleq \hat{\overline{\psi}}_{m,i}$.

**Proof:** Since $\overline{\Psi}_{m,i}(x) = \overline{\psi}_{m,i}x^{-\alpha}$ and $\hat{\Psi}_{m,i}(x) = \hat{\psi}_{m,i}x^{-\alpha}$, their inverse functions are given by $\overline{\Psi}_{m,i}^{-1}(x) = \overline{\psi}_{m,i}^{-1}(1/x^{\alpha})$ and $\hat{\Psi}_{m,i}^{-1}(x) = \hat{\psi}_{m,i}^{-1}(1/x^{\alpha})$, respectively. Substituting these explicit results of $\overline{\Psi}_{m,i}(x)$, $\overline{\Psi}_{m,i}^{-1}(x)$, $\hat{\Psi}_{m,i}(x)$ and $\hat{\Psi}_{m,i}^{-1}(x)$ into (5) and (7) results in (11) and (12).

With the results in Corollary 1, we can find the statistical properties of the biased pathloss of the associated BS and the association probability of each tier for any power-law pathloss-based association policies, such as nearest BS association (NBA), maximum mean received power association (MMPA), green cell association (GCA), etc. [8]–[10]. We will expound how to use Corollary 1 to analyze the coverage probability with the biased MMA model in Section III. Also, Corollary 1 can be applied to find the distribution of the cell load of the BSs in a particular tier as shown in the following subsection.

**Fading and Shadowing Gain Models.** Assume the BSs in the $m$th tier are equipped with $T_m$ transmit antennas and all users are equipped with a single antenna. We consider that all the channels from a BS to a user independently undergo small scale Rayleigh fading and large-scale lognormal shadowing with blockage effects. Consider the BSs in the $m$th tier are equipped with $T_m$ transmit antennas and each user is equipped with a single antenna for analytical simplicity. In other words, the channel from each BS to a user is a multiple-input-single-output (MISO) channel. The MISO channel gain from BS $X_m$ to its serving user can be written as

$$\tilde{H}_m \triangleq H_s G_s,$$

where $H_s$ denotes the small-scale channel fading gain and $G_s = \overline{G}_s[1 - \ell([X_s])] + \hat{G}_s[1 - \ell([X_s])]$ is the large-scale shadowing gain in which $\overline{G}_s$ and $\hat{G}_s$ denote the LOS and NLOS shadowing gains, respectively. Moreover, we assume that $H_s \sim (2T_m)^{-1}$ is a Chi-squared RV with $2T_m$ degrees of freedom, $\overline{G}_s \sim \ln N(0, \overline{\sigma}_g^2)$ and $\hat{G}_s \sim \ln N(0, \hat{\sigma}_g^2)$ are log-normal RVs if BS $X_s \in \Phi_m$. Similarly, the interference channel gain from BS $X_{m,i}$ to the typical user is written as

$$\tilde{H}_{m,i} = H_{m,i} G_{m,i}, \tag{14}$$

where $H_{m,i} \sim \exp(1)$ is an exponential RV with unit mean and variance $\sigma_h^2$ and $G_{m,i} = G_{m,i}[\ell([X_{m,i}])] + G_{m,i}[1 - \ell([X_{m,i}])]$ in which $\overline{G}_{m,i} \sim \ln N(0, \overline{\sigma}_g^2)$ and $\hat{G}_{m,i} \sim \ln N(0, \hat{\sigma}_g^2)$. Note that all $H_{m,i}$’s are independent for all $i \in \mathbb{N}_+ m \in \mathcal{M}$, and they are i.i.d. for the same tier.

**C. The SINR model for UHF and mmWave bands**

According to the generalized user association scheme in (2) and the power-law user association function designed in (10), the signal-to-interference plus noise power ratio (SINR) of the typical user can be written as

$$\text{SINR}_{s} = \frac{\tilde{H}_s P_{s}/L_s([X_s])}{I_{m}^s + W_s N_0 I(X_s \notin \Phi_M)}$$

for the interfering BSs are not beamformed to the typical user [11], [12].
where $W_u$ and $W_c$ denote the bandwidths for the UHF BSs and mmWave BSs, respectively. $L_s(\|X_s\|)$ is the pathloss function of BS $X_s$ as defined in (8), $P_s \in \{P_1, \ldots, P_M\}$ is the power of BS $X_s$, $H_s \in \{H_{m,i}\}_{m \in M, i \in \mathbb{N}_+}$ where $H_{m,i}$ is given in (14) is the channel gain of $X_{m,i}$, $N_0$ is the power spectrum density of the thermal noise, interferences $I_U^*$ and $I_L^*$ are given by

$$I_U^* = \sum_{m,i \in \Phi_M \setminus \{X_s\}} \frac{P_m H_{m,i}}{L_{m,i}(\|X_{m,i}\|)};$$

$$I_L^* = \sum_{M, i \in \Phi_M \setminus \{X_s\}} \frac{P_m H_{m,i}}{L_{m,i}(\|X_{m,i}\|)},$$

respectively.

The SINR model in (15) can be simplified to another low-complexity model by considering the practical signal propagation characteristics in the UHF and mmWave bands. In the UHF band, the interference usually dominates the received signal power so that the UHF BSs are interference-limited, whereas the received signal power in the mmWave Band is usually dominated by the noise power since the mmWave BSs need to overcome the high pathloss by beamforming to their users so that users usually have non-beamformed weak interference channels and have a non-negligible noise power due to the large bandwidth in the mmWave band [2], [13]. As such, the SINR in (15) can be accurately rewritten as

$$\gamma_s \triangleq \frac{P_s H_s}{I_U^* L_s(\|X_s\|)} 1(X_s \notin \Phi_M) + \frac{P_s H_s}{(I_c^* + W_c N_0) L_s(\|X_s\|)} 1(X_s \in \Phi_M)$$

by assuming $I_c^* \gg W_c N_0$ almost surely. Namely, we consider an SIR model in the UHF and an SINR model in the mmWave bands. Instead of using the SINR in (15), we will use the SINR model in (18) to analyze the coverage probability in the following section.

### III. ANALYSIS OF COVERAGE PROBABILITY

Suppose the SINR threshold for success decoding at each user is $\theta$. By using the SINR model in (18), the (downlink) coverage probability of a user in the mmWave HetNet can be defined as follows

$$p_{cov}(\theta) \triangleq \mathbb{P} [\gamma_s \geq \theta] = \sum_{m=1}^{M} \phi_m \mathbb{P} [\gamma_s \geq \theta | X_s \in \Phi_m].$$

Using $\gamma_s$ in (18) leads to $p_{cov}(\theta)$ explicitly given by

$$p_{cov}(\theta) = \sum_{m=1}^{M-1} \phi_m \left[ \frac{P_s H_s}{(I_c^* + W_c N_0) L_s(\|X_s\|)} \geq \theta \right] + \phi_M \left[ \frac{P_s H_s}{I_c^* L_s(\|X_s\|)} \geq \theta \right].$$

Since the distribution of $L_s(\|X_s\|)$ depends on how the user association function is designed, $p_{cov}(\theta)$ highly depends on the user association scheme. The user association signals emitted from the BSs usually undergo small-scale fading and large-scale shadowing whereas only the fading component in the signals usually can be averaged out at receivers. Hence, for the user association function (10) that characterizes the shadowing gain, the coverage probability in (20) can be explicitly found as shown in the following theorem.

**Theorem 2.** Consider the user association function in (10) that is able to characterize the channel shadowing gain of BS $X_{m,i}$ is designed as $\Psi_{m,i}(\|X_{m,i}\|) = \frac{\omega_{m} G_{m,i}}{L_{m,i}(\|X_{m,i}\|)}$ where $\omega_m$ is the constant bias for the tier-$m$ BSs. The coverage probability in (20) can be found as

$$p_{cov}(\theta) = \sum_{m=1}^{M-1} \phi_m B_m(\theta) + \phi_M B_M(\theta),$$

where $B_m(\theta) \triangleq \sum_{n=0}^{T_m} (-\theta)^n \frac{d^n}{d\theta^n} B_m(\theta)$, $B_M(\theta) \triangleq \sum_{n=0}^{T_M} (-\theta)^n \frac{d^n}{d\theta^n} B_M(\theta)$ and $\phi_m$ is given by

$$\phi_m = 2\pi \lambda_m \mathbb{E} \Psi_m^+ \left\{ \int_0^\infty e^{-\pi \sum_{k=1}^{M} \lambda_k A_k \phi_m^+ (x) x dx} \right\},$$

where $A_k \triangleq \int_0^\infty \mathbb{E} \left[ \frac{I_x}{G_{k,m}^+} \right] + \left[ \frac{\eta \beta_m \omega_m \eta_r}{\sum_{k=1}^{M} \lambda_k G_{k,m}^+} \right] + \left[ \frac{\eta_r}{\sum_{k=1}^{M} \lambda_k G_{k,m}^+} \right] \right\}$.

**Proof:** See Appendix B.

Theorem 2 reveals a few important implications. First, the coverage probability in (21) reflects how the coverage are much improved by adding more mmWave BSs that are equipped with a single transmit antenna, respectively. As a result, we can obviously see how much coverage probability in (21) is improved by adding more additional antennas. Third, since the NLOS and LOS channels are assumed to independently and statistically suffer different shadowing gains, the coverage probability is characterized by $m$ independent inhomogeneous PPPs whose intensities are
shown in (23), which is the main reason that \( B'_{m}(\theta) \) in (24) and \( B'_M(\theta) \) in (25) cannot be further simplified to a low-complexity result. However, we can consider some special cases, such as high penetration loss and noise-limited characteristics in the mmWave channels and a simpler shadowing and pathloss model in the UHF channels, that should be able to largely simplify \( B'_{m}(\theta) \) as well as \( B'_M(\theta) \).

IV. NUMERICAL RESULTS

In this section, we provide some numerical results to validate our analytical findings of the tier-\( m \) association probability in (22) and the coverage probability in (21). We consider two tiers in the HetNet and the first tier consists of the UHF macrocell BSs and the second tier consists of mmWave picocell BSs. The network parameters for simulation are: \( P_l = 20 \) W, \( P_d = 1 \) W, \( \theta = 1 \), \( \lambda_1 = 1 \times 10^{-6} \) (BSs/m\(^2\)), \( \beta = 5 \times 10^{-5} \) (blockages/m\(^2\)), \( \eta = 1.5 \) dB, log-normal LOS and NLOS shadowing variances \( \sigma^2_1 = \sigma^2_2 = 12 \) dB for the UHF macrocell BSs, LOS and NLOS pathloss exponents \( \sigma = 2.1, \tilde{\sigma} = 3.4 \) for the UHF macrocell BSs, LOS and NLOS pathloss exponents \( \sigma = 2.1, \tilde{\sigma} = \infty \) for the 73-GHz mmWave picocell BSs with very large penetration loss, and log-normal LOS and NLOS shadowing variances \( \sigma^2_1 = 15.6 \) dB, \( \sigma^2_2 = 21.8 \) dB for the mmWave picocell BSs. Also, users associate with the BS that provides the largest received power to them, i.e., they use the MMPA scheme with the user association function of BS \( X_{m,i} \) given by \( \Psi_{m,i}(|X_{m,i}|) = \frac{P_{m,m,i}}{\sum_{m=1}^{M} P_{m,m,i}} \).

The simulation results of the tier-\( m \) association probability are shown in Fig. 1 and we can see that the theoretical results of tier-\( m \) association probability perfectly coincides with their simulated results so that the derived \( \phi_m \) in (22) is correct. Also, since the penetration loss is assumed to be infinitely large, users that do not have LOS channels from any mmWave picocell BSs must associate with the macrocell BSs so that we need to densely deploy the mmWave BSs in order to make the majority of users associate with the mmWave picocell BSs. Otherwise, most traffic in the mmWave HetNet is still carried by macrocell BSs, which does not improve the network capacity since the large bandwidth resource in the mmWave band is not effectively utilized.

In Fig. 2, the simulation results of the coverage probabilities \( p_{cov}(\theta) \), \( B_1(\theta) \) and \( B_2(\theta) \) are demonstrated. We can observe that the (overall) coverage probability \( p_{cov}(\theta) \) is very high since all BSs are equipped with multiple antennas. The coverage probability contributed by the mmWave picocells initially slightly increase and then slightly decreases (so that \( p_{cov}(\theta) \) also slightly reduces) as more and more mmWave picocells are deployed. This is because initially deploying more picocells can increase the SINR and then the SINR cannot be effectively increased due to blockages and large interference. This phenomenon provides us a good insight about how to optimally and economically deploy the mmWave small BSs to enhance the coverage probability.

V. CONCLUDING REMARKS

In an urban area, the characteristics of wireless channels are seriously affected by the blockages, especially the channels in the mmWave band. To completely characterize LOS and NLOS channels induced by the blockages, in this work we develop a very general framework based on stochastic geometry to evaluate the coverage performance in a mmWave HetNet. The coverage probability with a generalized user association scheme is derived for a downlink MISO channel in an \( M \)-tier HetNet consisting of one tier of mmWave small cell BSs and \( M - 1 \) tiers of the UHF BSs. We show that the coverage probability can be effectively enhanced in this HetNet if compared with the cellular network consisting of a single tier of mmWave small cell BSs whose coverage probability can be seriously weakened by the dense blockages in an urban area.
expressed as
\[ A_m(z) = \mathbb{E}_L \left\{ \int_0^\infty 2 \left( \ell(r) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1}(z) \right] + (1 - \ell(r)) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1}(z) \right] \right) r dr \right\}. \]

Thus, \( A_m \circ \Psi_m(x) \triangleq A_m(\Psi_m(x)) \) and \( A_k \circ \Psi_m^i(x) \triangleq A_k(\Psi_m^i(x)) \) can be found as shown in the following:
\[
A_m \circ \Psi_m(x) = \mathbb{E}_L \left\{ \int_0^\infty 2 \left( \ell(r) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1} \circ \Psi_m(x) \right] + (1 - \ell(r)) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1} \circ \Psi_m(x) \right] \right) r dr \right\} = x^2,
\]
\[
A_k \circ \Psi_m^i(x) = \mathbb{E}_L \left\{ \int_0^\infty 2 \left( \ell(r) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1} \circ \Psi_m^i(x) \right] + (1 - \ell(r)) \mathbb{P} \left[ r < \overline{\Psi}_m^{-1} \circ \Psi_m^i(x) \right] \right) r dr \right\},
\]
which can be shown to equal to the result in (7). Next, we know that probability \( \phi_m \) can be explicitly defined as
\[
\phi_m = \mathbb{P} \left[ \sup_{m,i \in \Phi_m} \Psi_m, i(||X_m,i||) > \sup_{k,i \in \Phi_m} \Psi_k, i(||X_k,i||) \right] = \int_0^\infty \mathbb{P} \left[ \sup_{k,i \in \Phi_m} \Psi_k, i(||X_k,i||) < z \right] dF_{Z_m}(z),
\]
where RV \( Z_m \triangleq \sup_{m,i \in \Phi_m} \Psi_m, i(||X_m,i||) \). The CDF of \( Z_m \) given by
\[
F_{Z_m}(z) = \exp \left( -\pi \lambda_m A_m(z) \right), \tag{26}
\]
which can be inferred from (4) with only one PPP. According to (4), we also know
\[
\mathbb{P} \left[ \sup_{k,i \in \Phi \setminus \Phi_m} \Psi_k, i(||X_k,i||) < z \right] = e^{-\pi \sum_{k \in M \setminus \{m\}} \lambda_k A_k(z)}. \tag{27}
\]
Thus, substituting (26) and (27) into \( \phi_m \) given above leads to
\[
\phi_m = \int_0^\infty \exp \left( -\pi \sum_{k \in M \setminus \{m\}} \lambda_k A_k(z) \right) dF_{Z_m}(z) = \mathbb{E} \left\{ \int_0^\infty \exp \left( -\pi \sum_{k \in M \setminus \{m\}} \lambda_k A_k(\Psi_m^i(x)) \right) dF_m(\Psi_m^i(x)) \right\}
\tag{28}
\]
where (e) follows from \( dF_m(\Psi_m^i(x)) = 2\pi x \exp(-\pi \lambda_m A_m \circ \Psi_m^i(x)) dx \) for given \( \Psi_m^i(x) \). Hence, \( \phi_m \) in (6) is obtained.

B. Proof of Theorem 2

Since \( \Psi_m, i(x) = \omega_m G, m, i x^{-\alpha_m}, i \), we know \( \overline{\psi}_m, i = \omega_m \overline{G}, m, i \) and \( \psi_m, i = \omega_m G, m, i \). According to (12), we can have
\[
A_k \circ \Psi_m^i(x) = \text{given in Theorem 2 due to } E[\tau_k^{2/\alpha}] = e^{\frac{\sqrt{2}}{\alpha}}.
\]
Then substituting this into (6) leads to $\phi_m$ in (22). For $X_s \in \Phi_m$ and $m \in \{1, 2, \ldots, M-1\}$, the coverage probability can be found by

$$
P \left[ \frac{P_a H_a}{I_0^m L_s(\|X_s\|)} \geq \theta \right] = P \left[ H_0^m(\theta) \geq \theta I_0^m L_s(\|X_s\|) / P_a G_s \right] = \sum_{n=0}^{T_{m-1}} \frac{(-\theta)^n}{n!} \frac{d^n}{d\theta^n} \mathbb{E} \left[ \exp \left( -\theta \omega^m G_s \|X_s\| / \sum_{L \in \mathbb{Z}(X_s)} \right) \right],$$

where $\omega^m G_s / L_s(\|X_s\|) \triangleq \sup_{x \in \Phi \cap \mathbb{Z}(X_s)} \omega^m G_s$ and $\omega^m \in \{\omega^m(1, m)\}$ is the user association bias used by BS $X_s$. According to (11), the probability of $\frac{G_s}{L_s(\|X_s\|)} \leq \frac{1}{L_s(x)}(x)$ can be written as

$$
P \left[ \frac{\omega^m G_s}{L_s(\|X_s\|)} \leq \frac{1}{L_s(x)}(x) \right] = P \left[ \frac{\|X_s\|}{\omega^m G_s} \geq x \right] = \exp \left\{ -\pi \sum_{m=1}^M A_m(x) \right\},$$

where $A_m(x)$ is found as

$$A_m(x) = x^{2} \left( \mathbb{E} \left[ \int e^{(\omega^m G_m)^{1/\alpha}} 2 \pi e^{-(\theta/\omega^m G_m)^{1/\alpha}} \right] \right) + \frac{\omega^m_n}{\omega^m G_m} \mathbb{E} \left[ G_m^2 \right]$$

because we can let $\Psi_{m,i}(x) = (\omega^m G_m)^{1/\alpha} x^{\alpha_i - 1} \Psi_{m,i}(x)$ and $\tilde{\lambda}_m(r)$ is found as $x(\omega^m G_m)^{1/\alpha}$ and then substitute them into (5). Thus, it follows that

$$P \left[ \frac{\|X_s\|}{\omega^m G_s} \geq x \right] = \exp \left\{ -2\pi \int_0^x \sum_{m=1}^M \tilde{\lambda}_m(r) r dr \right\},$$

where $\tilde{\lambda}_m(r)$ is given in (23) and this manifests that $X_s(\omega^m G_s)^{-1/\alpha}$ can be viewed as the point of an inhomogeneous PPP of intensity $\lambda(r)$ nearest to the typical user.

Accordingly, we have

$$\mathbb{E} \left[ e^{r \omega^m G_m / P_a G_s} \right] \approx \mathbb{E} \left[ e^{-\sum_{m=1}^M \Phi_m(1) / \hat{\Phi}_m} \right],$$

in which $\Phi_m(1) = \hat{\Phi}_m$, $\hat{\Phi}_m$ is an inhomogeneous PPP of intensity $\hat{\lambda}_m(r)$ and $\hat{X}_s$ is the point in $\hat{\Phi}_m$ nearest to the typical user. The approximation is made by letting the pathloss exponents of all $\hat{X}_s$ be equal to the exponent of $\hat{X}_s$ in order to facilitate the following derivations. Therefore, we can have

$$\mathbb{E} \left[ \exp \left( -\frac{\theta \omega^m}{P_a G_s} \sum_{m=1}^M \frac{P_m H_m \|\hat{X}_s\|^{\alpha_m}}{\omega^m \|\hat{X}_s\|^{\alpha_m}} \right) \right] = \prod_{k=1}^{M-1} \sum_{\hat{X}_s \in \hat{\Phi}_k} \frac{1}{\|\hat{X}_s\|^{\alpha_k}} \partial (\hat{X}_s) \times \mathbb{E} \left[ e^{-2\pi \sum_{k=1}^{M-1} \int_0^{\infty} \hat{\lambda}_k(f) \left( \frac{e^{-2\pi f \|\hat{X}_s\|^2 / r^2}}{2\pi f} + e^{2\pi f \|\hat{X}_s\|^2 / r^2} \right) \mathbb{E} \left[ e^{-2\pi f \|\hat{X}_s\|^2 / r^2} \right] \right]$$

where (*) follows from the probability generating functional (PGFL) of $m$ independent inhomogeneous PPPs [4] and $\|\hat{X}_s\| = r$ where $P[\alpha_s = \alpha] = 1 - \exp(-\eta r/\theta)$. The coverage probability can be shown as

$$P \left[ \frac{P_a H_a}{I_0^m L_s(\|X_s\|)} \geq \theta \right] = \mathbb{E} \left[ \exp \left( -\frac{\theta I_0^m L_s(\|X_s\|)}{P_a G_s} \right) \right],$$

and using the same approach of showing $B_m' \triangleq \mathbb{E} \left[ e^{-2\pi \sum_{k=1}^{M-1} \int_0^{\infty} \hat{\lambda}_k(f) \left( \frac{e^{-2\pi f \|\hat{X}_s\|^2 / r^2}}{2\pi f} + e^{2\pi f \|\hat{X}_s\|^2 / r^2} \right) \mathbb{E} \left[ e^{-2\pi f \|\hat{X}_s\|^2 / r^2} \right] \right]$$

The proof is complete.

REFERENCES


