Application of Array Signal Processing Techniques to the Design of Rake Receivers

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OUTLINE

1. Array Signal Processing Concept
2. Blind Channel Estimation Using Array Signal Processing Techniques
3. Principal Component Combiner with Differential Decoding for DPSK Signals
Array Signal Processing
Concepts
Direction of Arrival Estimation Problem
The Mathematical Model for the DOA Estimation Problem

The signal received at the \( m \)-th antenna is

\[
    r_m(t) = \sum_{k=1}^{K} s_k(t) e^{-j2\pi m \frac{d}{\lambda} \sin(\theta_k)} + n_m(t), \quad m = 0, 1, \ldots, M - 1.
\]

where \( \theta_k \) is the direction of arrival of user \( k \). It can be written in vector form

\[
    \begin{bmatrix}
    r_0(t) \\
    r_1(t) \\
    \vdots \\
    r_{M-1}(t)
    \end{bmatrix}
    = \sum_{k=1}^{K} \begin{bmatrix}
    1 \\
    e^{-j2\pi \frac{d}{\lambda} \sin(\theta_k)} \\
    \vdots \\
    e^{-j2\pi (M-1) \frac{d}{\lambda} \sin(\theta_k)}
    \end{bmatrix}
    \begin{bmatrix}
    s_k(t) \\
    n_0(t) \\
    n_1(t) \\
    \vdots \\
    n_{M-1}(t)
    \end{bmatrix}.
\]
Direction of Arrival Estimation Methods from Array Signal Processing

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional Method</td>
<td>$\hat{\phi} = \arg \max a^H(\phi) \hat{R}a(\phi)$</td>
</tr>
<tr>
<td>Constrained Method</td>
<td>$\hat{\phi} = \arg \min a(\phi)^H \hat{R}^{-1}a(\phi)$</td>
</tr>
<tr>
<td>Subspace Method</td>
<td>$\hat{\phi} = \arg \min a^H(\phi) \hat{V}_n \hat{V}_n^H a(\phi)$</td>
</tr>
</tbody>
</table>

where

- $a(\phi)$ is the steering vector
- $\hat{R}$ is the sample covariance matrix
- $\hat{V}_n$ contains the vectors in the noise subspace of $\hat{R}_n$
Blind Channel Estimation Using Array Signal Processing Techniques
Multipath Fading for the Code Division Multiple Access System

path 0

path 1

path 2

t_2

path L-1

t_{L-1}
Discrete-Time Channel Model for the Desired User

The effective signature vector \( \mathbf{h} \triangleq [h(0) \ h(1) \ \cdots \ h(L_c + q - 1)]^T \)

\[
\mathbf{h} = \begin{bmatrix}
    c(0) & \cdots & 0_q \\
    c(1) & \cdots & c(0) \\
    \vdots & \ddots & \vdots \\
    c(L_c - 1) & & \vdots \\
    0_q & \cdots & c(L_c - 1)
\end{bmatrix} \begin{bmatrix}
    g(0) \\
    g(1) \\
    \vdots \\
    g(q)
\end{bmatrix}
\]

where

- \( 0_q \) is a zero vector of dimension \( q \times 1 \)
- \( g(i), i = 0, 1, \cdots, q - 1 \) be the baseband equivalent impulse response
- \( c(0), c(1), \cdots, c(L_c - 1) \) be the spreading code of the desired user
Asynchronous CDMA Channel Model at the Receiver

Assume perfect synchronization and chip-rate sampling.

\[ \mathbf{r}(n) \triangleq \begin{bmatrix} r(nL_c) \\ r(nL_c + 1) \\ \vdots \\ r(nL_c + L_c + q - 1) \end{bmatrix} = \mathbf{h} b(n) + \mathbf{w}(n) + \mathbf{i}(n) + \mathbf{v}(n) \]

where

- \( \mathbf{w}(n) \) is the intersymbol interference
- \( \mathbf{i}(n) \) is the multiuser interference
- \( \mathbf{v}(n) \) is the additive noise
Asynchronous CDMA Channel Model at the Receiver

• The intersymbol interference

\[ w(n) \triangleq \begin{bmatrix} h(L_c) \\ \vdots \\ h(L_c + q - 1) \\ 0_{L_c} \end{bmatrix} b(n - 1) + \begin{bmatrix} 0_{L_c} \\ h(0) \\ \vdots \\ h(q - 1) \end{bmatrix} b(n + 1) \]

• The multiuser interference \( i(n) \) is comprised of components contributed by all other multiusers in the same system. Each of the multiusers is modelled in the same manner as the desired user but with a different delay and a different multipath channel.

• The additive noise \( v(n) \) is assumed to be white Gaussian with covariance matrix \( \sigma^2 I \).
The Conventional Method for Blind Channel Estimation

In case that a single user exist and that the delay spread is small, the received signal is reduced to

\[ r_{\text{conv}}(n) = h \cdot b(n) + v(n) \]

The solution is

\[
\hat{h}_{\text{conv}} = \arg \max_f E\{|f^H r_{\text{conv}}(n)|^2\} \text{ subject to } ||f|| = 1
\]

\[
= \arg \max_f f^H R f \text{ subject to } ||f|| = 1
\]

\[
= \arg \max_f \frac{f^H R f}{f^H f}
\]
The Conventional Method for Blind Channel Estimation

The conventional method for blind channel estimation is

\[
\hat{g}_{\text{conv},1} = \arg \max_g \frac{g^H C^H \hat{R} C g}{g^H C^H C g}
\]

which indicates that \( \hat{g}_{\text{conv},1} \) is the normalized generalized eigenvector of \((C^H \hat{R} C, C^H C)\) that is associated with the largest eigenvalue, or

\[
\hat{g}_{\text{conv},2} = \arg \max_g \frac{g^H C^H \hat{R} C g}{g^H g}
\]

which indicates that \( \hat{g}_{\text{conv},2} \) is the normalized eigenvector of \( C^H \hat{R} C \) that is associated with the largest eigenvalue.
The Constrained Method for Blind Channel Estimation

The constrained method for blind channel estimation is

\[ \hat{\mathbf{g}}_{\text{capon},1} = \arg \min_{\mathbf{g}} \frac{\mathbf{g}^H \hat{\mathbf{R}}^{-1} \mathbf{C} \mathbf{g}}{\mathbf{g}^H \mathbf{C}^H \mathbf{C} \mathbf{g}} \]

which indicates that \( \hat{\mathbf{g}}_{\text{capon},1} \) is the normalized generalized eigenvector of \( (\mathbf{C}^H \hat{\mathbf{R}}^{-1} \mathbf{C}, \mathbf{C}^H \mathbf{C}) \) that is associated with the smallest eigenvalue. A different selection of the constrained method for \( \mathbf{g} \) is

\[ \hat{\mathbf{g}}_{\text{capon},2} = \arg \min_{\mathbf{g}} \frac{\mathbf{g}^H \hat{\mathbf{R}}^{-1} \mathbf{C} \mathbf{g}}{\mathbf{g}^H \mathbf{g}} \]

which indicates that \( \hat{\mathbf{g}}_{\text{capon},2} \) is the normalized eigenvector of \( \mathbf{C}^H \hat{\mathbf{R}}^{-1} \mathbf{C} \) that is associated with the smallest eigenvalue.
The Subspace Method for Blind Channel Estimation

The subspace method for blind channel estimation is

$$\hat{g}_{sub,1} = \arg \min_g \frac{g^H C^H \hat{E}_n \hat{E}_n^H C g}{g^H C^H C g}$$

which indicates that $\hat{g}_{sub,1}$ is the normalized generalized eigenvector of $(C^H \hat{E}_n \hat{E}_n^H C, C^H C)$ that is associated with the smallest eigenvalue. Similarly, we can select $g$ as

$$\hat{g}_{sub,2} = \arg \min_g \frac{g^H C^H \hat{E}_n \hat{E}_n^H C g}{g^H g}$$

which indicates that $\hat{g}_{sub,2}$ is the normalized eigenvector of $C^H \hat{E}_n \hat{E}_n^H C$ that is associated with the smallest eigenvalue.
Simulation Environment

- Pulse shaping function: root-raised cosine function with rolloff factor 0.5
- Gold code of length 31
- Four-ray multipath channel with relative path gains 0, -3, -6, and -9 dB
- Differential BPSK signals
- Sample covariance is estimated by $\hat{R} = \sum_{n=1}^{100} r(n)r^H(n)/100$
- Number of Monte Carlo runs: 1000
- Minimum mean-squared error detector $\hat{f} = \hat{E}_s\hat{D}_s^{-1}\hat{E}_s^H C \hat{g}$

where

$$\hat{R} = \begin{bmatrix} \hat{E}_s & \hat{E}_n \end{bmatrix} \begin{bmatrix} \hat{D}_s & \hat{D}_n \end{bmatrix}^H \begin{bmatrix} \hat{E}_s & \hat{E}_n \end{bmatrix}^H$$
Comparisons of Blind Channel Estimation Methods Under the 4-ray Channel for Different Signal-to-Noise Ratios

![Graph showing Bit Error Rate vs. Signal to Noise Ratio for 10 users, 4-ray channel, DBPSK. The graph compares Conventional Method, Constrained Method, Subspace Method, Torlak–Xu Method (SF=2), Torlak–Xu Method (SF=3), and True Channel Information.](image-url)
Comparisons of Blind Channel Estimation Methods Under the 4-ray Channel for Different Number of Interferers

SNR is 10, 4-ray channel, DBPSK

- Conventional Method
- Constrained Method
- Subspace Method
- Torlak-Xu Method (SF=2)
- Torlak-Xu Method (SF=3)
- True Channel Information
Comparisons of Blind Channel Estimation Methods Under the
4-ray Channel for a Strong Interferer at Different Interference
to the Desired User Power Ratio

SNR is 10 dB, 10 users, 4-ray channel, DBPSK
Comparisons of Blind Channel Estimation Methods Under Different Number of Multipath Components

SNR is 10 dB, 10 users, 4-ray channel, DBPSK

- Conventional Method
- Constrained Method
- Subspace Method
- Torlak–Xu Method (SF=2)
- Torlak–Xu Method (SF=3)
- True Channel Information
MSE Comparisons of Blind Channel Estimation Methods Under Different Intercell Interference to the Desired Signal Ratio

SNR is 10 dB, 5 users, 4-ray channel, DBPSK

- + : Conventional Method
- x : Constrained Method
- o : Subspace Method
- * : Torlak–Xu Method (SF=2)
- △ : Torlak–Xu Method (SF=3)
## Comparisons of the 3 Blind Channel Estimation Methods

<table>
<thead>
<tr>
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<th>Subspace Method</th>
</tr>
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<tbody>
<tr>
<td><strong>Performance</strong></td>
<td>Poor</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td><strong>Near-Far Resistance</strong></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Robust to Intercell Interference</strong></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Complexity</strong></td>
<td>0</td>
<td>$O((L_c + q)^2)$</td>
<td>$O((L_c + q)^3)$</td>
</tr>
<tr>
<td><strong>Adaptation Complexity</strong></td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>
Principal Component Combiner with Differential Decoding for DPSK Signals
Problem of Interest

The signal received at the $i$-th diversity channel is

$$r_i(k) = \sqrt{E_s} c_i e^{j\phi(k)} + v_i(k), \quad i = 1, 2, \cdots, L$$

where
- $c_i$ is the channel gain
- $\phi(k) = \phi(k - 1) + \Delta\phi(k)$ carries the $M$-ary DPSK information
- $v_i(k)$ is the zero-mean white Gaussian noise with variance $N_0$

The above model can be written in vector form

$$\mathbf{r}(k) = \sqrt{E_s} \mathbf{c} e^{j\phi(k)} + \mathbf{v}(k)$$
The Differential Decoding with Equal Gain Combiner

\[ r_1(k) \]
\[ Z^{-1} \]
\[ (\cdot)^* \]
\[ r_2(k) \]
\[ Z^{-1} \]
\[ (\cdot)^* \]
\[ r_L(k) \]
\[ Z^{-1} \]
\[ (\cdot)^* \]

Differential Detector

Equal Gain Combiner

Decision
Principal Component Combiner with Differential Decoding

\[ r_1(k) \quad r_2(k) \quad \ldots \quad r_L(k) \]

Principal Component Combiner

\[ w_1^* \quad w_2^* \quad \ldots \quad w_L^* \]

Differential Detector

\[ z(k) \]

Decision

\[ y(k) \]

\[ Z^{-1} \quad (\cdot)^* \]
Principal Component Combiner with Differential Decoding

The PCC-DD determine the combining weight as

$$w_P = \arg \max_w E\{\|w^H r(k)\|^2 / \|w\|^2\}$$

$$= \arg \max_w w^H R_r w / \|w\|^2$$

where the covariance matrix is

$$R_r = E_{scc}^H + N_0 I$$

Theoretically,

$$w_P = e^{j\theta} c / \|c\|$$
Recall that the BEP of the binary DPSK signal over AWGN channel is

\[ P_2(\gamma_s) = \frac{1}{2} e^{-\gamma_s} \]

where the instantaneous SNR per symbol

\[ \gamma_s \triangleq \frac{E_s}{\| c \|^2 / N_0} \]

Recall that the instantaneous SNR is Chi-Squared distributed with density function

\[ p(\gamma_s) = \frac{1}{(L - 1)! \bar{\gamma}_c^L} \gamma_s^{L-1} e^{-\gamma_s / \bar{\gamma}_c} \]

where the average SNR per channel is defined by

\[ \bar{\gamma}_c \triangleq \frac{E_s}{N_0} E\{|c_i|^2\} \]
The exact BEP of binary DPSK signals for the PCC-DD is

\[ P_2 = \int_0^\infty P_2(\gamma_s) p(\gamma_s) \, d\gamma_s \]

\[ = \frac{1}{2} \frac{1}{(1 + \bar{\gamma}_c)^L} \int_0^\infty \frac{\gamma_s^{L-1} e^{-\gamma_s/(\bar{\gamma}_c/(1+\bar{\gamma}_c))}}{(L - 1)!((\bar{\gamma}_c/(1 + \bar{\gamma}_c))^L} \, d\gamma_s \]

\[ = \frac{1}{2} \frac{1}{(1 + \bar{\gamma}_c)^L} \]

Note that the last equality follows from the fact that the integral in the second line is equal to unity, because integral of the probability density function of a chi-square distributed random variable is unity.
BEP of $M$-ary DPSK Signals over Rayleigh Fading Channels

The approximate BEP of $M$-ary DPSK signals for the PCC-DD is

$$P_M \approx \frac{2}{\log_2 M} \left( \frac{1 - \mu}{2} \right)^L \sum_{k=0}^{L-1} \binom{L - 1 + k}{k} \left( \frac{1 + \mu}{2} \right)^k$$

where

$$\mu = \sqrt{\frac{1}{2} \sin^2 \frac{\pi}{M} \tilde{\gamma}_c} \frac{1 + \frac{1}{2} \sin^2 \frac{\pi}{M} \tilde{\gamma}_c}{1 + \frac{1}{2} \sin^2 \frac{\pi}{M} \tilde{\gamma}_c}$$
Adaptive Calculation of the Principal Component Combining Weight

Use the rank-1 subspace tracker, e.g., Karhunen’s method

% Initialization: $\mathbf{u}(0) = [1, 0, \cdots, 0]^T$
% $\lambda(0) = 1$, $\beta = 0.99$
% As time increases, $k = 1, 2, 3, \cdots$

Complexity

$\xi = \mathbf{r}^H(k)\mathbf{u}(k - 1)$
$\alpha = \xi/\beta/\lambda(k - 1)$
$\mathbf{u}(k) = \mathbf{u}(k - 1) + \alpha \mathbf{r}(k)$
$\mathbf{u}(k) = \mathbf{u}(k)/\|\mathbf{u}(k)\|$
$\lambda(k) = \beta\lambda(k - 1) + |\xi|^2$

% Go to $k = k + 1$

$L$
$2$
$L$
$2L$
$2$
BEP Comparisons of 2-DPSK Signals for PCC-DD and DD-EGC over Rayleigh Fading Channels
BEP Comparisons of 4-DPSK Signals for PCC-DD and DD-EGC over Rayleigh Fading Channels
BEP Comparisons of 2-, 4-, and 8-DPSK Signals for PCC-DD over Rayleigh Fading Channels
Conclusions

- The exact BEP of binary DPSK signals for PCC-DD is available.

- The approximate BEPs of $M$-ary DPSK ($M \geq 4$) signals for PCC-DD are derived.

- For binary DPSK signals, the PCC-DD outperforms the DD-EGC by 0.9, 1.5, and 2.3 dB when the number of diversity channels is 2, 4, and 8, respectively.

- For $M$-ary DPSK ($M \geq 4$) signals, the PCC-DD outperforms the DD-EGC when the average SNR per bit is low, but performs approximately the same as the DD-EGC when the average SNR is high.

- The proposed PCC-DD needs $6L$ complex-valued multiplications in each iteration to update its principle combining weight vector.
References


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