Cost Benefit Analysis of Series Systems with Cold Standby Components and a Repairable Service Station

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Abstract: This paper studies the cost benefit analysis of series systems with cold standby components and a repairable service station. The service times and the failure times of the primary components are assumed exponentially distributed. The breakdown times and the repair times of the service station are also assumed exponentially distributed. We develop the explicit expressions for the mean time to failure (or \( MTTF \)) and the steady-state availability (or \( \bar{A}_T(\infty) \)) for three configurations and perform comparisons. Under the cost/benefit (\( C/B \)) criterion, comparisons are made for specific values of distribution parameters and of the costs of the components. For all three configurations, the configurations are ranked based on: \( MTTF \), \( \bar{A}_T(\infty) \), and \( C/B \) where \( B \) is either \( MTTF \) or \( \bar{A}_T(\infty) \).

Keywords: Availability, cold standbys, cost/benefit, mean time to failure, repairable service station, series system.

1. Introduction

In the literature of queueing problems, most of the papers study the series systems with standby components where the service station has never broken down. However, in real-life situations we often encounter cases where the service station may break down and can be repaired. A repairable service station means that the service station is typically subject to unpredictable breakdowns and can be repaired. The mean time to failure (or \( MTTF \)) and the steady-state availability (or \( \bar{A}_T(\infty) \)) have widely been analyzed in the literature because of their prevalence in power plants, manufacturing systems, and industrial systems. Maintaining a high or required level of reliability and/or availability is often an essential requisite. The standby component is called a ‘cold standby’ if its failure rate is zero. Primary components can be considered to be repairable.

We deal with the \( MTTF \), the \( \bar{A}_T(\infty) \), and the cost/benefit analysis of three different series system configurations with cold standby components and a repairable service station. These three configurations are compared based on their \( MTTF \), their \( \bar{A}_T(\infty) \), and their \( C/B \). Cost is considered to be a size-proportional cost for the Primary and cold standby components. Benefit is divided into two categories according to whether the measure utilized is the \( MTTF \) or the \( \bar{A}_T(\infty) \).

Gaikowsky et al. [2] and Wang and Pearn [9] examined the series systems with cold standby components and warm standby components, respectively, where the service station has never failed. Wang and Kuo [7] investigated the cost and probabilistic analysis of series systems with mixed standby components. Several queueing systems with a repairable
service station (server or repairman) have been studied by several researchers such as Cao [1], Li et al. [3], Tang [4], and Wang [5]. However, such systems can also be regarded as reliability models. Wang and Ke [6] studied the probability analysis of a repairable system with warm standbys plus balking and reneging for which no cost-benefit analysis is considered. Recently, Wang et al. [8] proposed cost benefit analysis of series systems with warm standby components and general repair time in which the server is not subject to breakdowns. The problem considered in this paper is more general than the works of Galikowsky et al. [2]. The purpose of this paper is threefold. The first purpose is to develop the explicit expressions for the $MTTF_i$ and the $A_{r_i}(\infty)$, for configuration $i$, where $i = 1, 2, 3$. The second purpose is to compare these configurations in terms of their $MTTF_i$, their $A_{r_i}(\infty)$, and their cost / benefit ratio $C_i/B_i$. The third one is to rank three configurations for the $MTTF$, the $A_{r_i}(\infty)$, and the $C_i/B_i$ based on assumed numerical values given to the system parameters, as well as to the costs of the components.

2. Description of the System

For the sake of discussion, we consider the requirements of a 30 MW power plant. We assume that generators are available in units of 30 MW, 15 MW and 10 MW. Let us assume that all switchover times are instantaneous and switching is perfect, e.g. never fails and never does any damage. Primary components can be considered to be repairable. Each of the primary components fails independently of the state of the others and has an exponential failure distribution with parameter $\lambda$. Whenever one of these components fails, it is immediately replaced by a cold standby component if one is available. Only one primary component can be served at a time. Whenever a primary component fails, it is immediately sent to a service station where it is served in order of breakdowns, with identical service rate $\mu$. Service time distribution of the components is assumed to be exponentially distributed. Once a component is served, it is as good as new. There is a single service station which may break down only when the service station is serving a component. Once the service station breaks down, the service station enters a break down state and a failed component must wait until the service station is repaired. The service station has an exponential failure distribution with rate $\alpha$. Whenever a service station breaks down, it is immediately repaired at a repair rate $\beta$. Repair time distribution of the service station is assumed to be exponentially distributed. Service is allowed to be interrupted if the station breaks down, and the station is immediately repaired. As soon as the repair of a service station is completed, the service station enters a working state and continues to serve a failed component immediately. After the service station is repaired, it is as good as new.

We consider three configurations as follows: The first configuration is a serial system of one primary 30 MW component with one cold standby 30 MW component (Figure 1). The second configuration is a serial system of two primary 15 MW components and one cold standby 15 MW component (Figure 2). The last configuration is a serial system of three primary 10 MW components with two interchangeable cold standby 10 MW components. (Figure 3). Each standby unit can replace either one of the failed components.

Cost-Benefit Ratio

We assume that the size-proportional costs for the primary components and cold standby components are given in Table 1. With this, we calculate the costs for each configuration $i (i = 1, 2, 3)$ shown in Table 2. Let $C_i$ be the cost of the configuration $i$, and $B_i$ be the benefit of the configuration $i$, where $B_i$ may either be the $MTTF_i$ (for system reliability), or the $A_{r_i}(\infty)$ (for system availability), where $i = 1, 2, 3$. 
Figure 1. Configuration 1: one primary 30 MW component and one cold standby 30 MW component.

Figure 2. Configuration 2: two primary 15 MW components and one cold standby 15 MW component.

Figure 3. Configuration 3: three primary 10 MW components and two interchangeable 10 MW cold standby components.
Table 1. The size-proportional cost for the primary and cold standby components.

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary 30 MW</td>
<td>30 \times 10^6</td>
</tr>
<tr>
<td>Primary 15 MW</td>
<td>15 \times 10^6</td>
</tr>
<tr>
<td>Primary 10 MW</td>
<td>10 \times 10^6</td>
</tr>
<tr>
<td>cold standby 30 MW</td>
<td>18 \times 10^6</td>
</tr>
<tr>
<td>cold standby 15 MW</td>
<td>9 \times 10^6</td>
</tr>
<tr>
<td>cold standby 10 MW</td>
<td>6 \times 10^6</td>
</tr>
</tbody>
</table>

Table 2. The costs for each configuration, $i$, $i = 1, 2, 3$.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Cost (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
<td>48 \times 10^6</td>
</tr>
<tr>
<td>Configuration 2</td>
<td>39 \times 10^6</td>
</tr>
<tr>
<td>Configuration 3</td>
<td>42 \times 10^6</td>
</tr>
</tbody>
</table>

3. Reliability Analysis

3.1. Calculations for Configuration 1

For configuration 1, let $P_n(t)$ be the probability that at time $t$ there are $n$ components working in the system when the service station is up and $Q_n(t)$ be the probability that at time $t$ there are $n$ components working in the system when the service station is broken down, where $t \geq 0$. If we let $P(t)$, and $Q(t)$ denote the probability row vector at time $t$, then the initial conditions for this problem are stated as follows:

**Initial conditions:**

The system has exactly 2 components working initially and the service station is up, i.e., $P_2(0) = 1$, and all other initial probabilities are zero.

$$P(0) = [P_2(0), P_1(0), P_0(0)] = [1, 0, 0], Q(0) = [Q_2(0), Q_1(0), Q_0(0)] = [0, 0, 0]$$

Omitting the argument $t$ in $P_n(t)$ and $Q_n(t)$ so that $P_n(t) = P_n$ and $Q_n(t) = Q_n$, we obtain the following differential equations:

$$\frac{dP_2}{dt} = -(\lambda + \alpha)P_2 + \mu P_1 + \beta Q_2,$$

$$\frac{dP_1}{dt} = \lambda P_2 - (\lambda + \mu + \alpha) P_1 + \beta Q_1,$$

$$\frac{dP_0}{dt} = \lambda P_1,$$

$$\frac{dQ_2}{dt} = \alpha P_2 - (\lambda + \beta) Q_2,$$

$$\frac{dQ_1}{dt} = \lambda P_2 - (\lambda + \mu + \alpha) Q_1 + \beta P_1,$$

$$\frac{dQ_0}{dt} = \lambda P_1.$$
\[ \frac{dQ_1}{dt} = \alpha P_1 + \lambda Q_2 - (\lambda + \beta)Q_1, \]
\[ \frac{dQ_0}{dt} = \lambda Q_1. \]

Let \( \mathbf{W} = [\mathbf{P}, \mathbf{Q}] \) denote the probability row vector. The above equations can be written in matrix form as

\[ \dot{\mathbf{W}} = \mathbf{W} \mathbf{B}_1, \tag{1} \]

where

\[
\mathbf{B}_1 = \begin{pmatrix}
-\lambda - \alpha & \lambda & 0 & \alpha & 0 & 0 \\
\mu & -\lambda - \mu - \alpha & \lambda & 0 & \alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\lambda - \beta & \lambda & 0 \\
0 & \beta & 0 & 0 & -\lambda - \beta & \lambda \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
\]

It is extremely difficult to develop the transient solutions. The simple procedure is provided to develop the explicit expression for the \( \text{MTTF}_1 \). We delete the third row and column, and the last row and column of matrix \( \mathbf{B}_1 \) for the absorbing state (s) which yields a new matrix, say \( \mathbf{A} \). The expected times to reach an absorbing state is obtained from

\[ E[\mathbf{T}_{\mathbf{W}(0)\rightarrow (\text{absorbing})}] = \mathbf{W}(0)(-\mathbf{A}^{-1}) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \tag{2} \]

where \( \mathbf{W}(0) = [\mathbf{P}(0), \mathbf{Q}(0)] = [1,0,0,0], \) and

\[
\mathbf{A} = \begin{pmatrix}
-\lambda - \alpha & \lambda & \alpha & 0 \\
\mu & -\lambda - \mu - \alpha & 0 & \alpha \\
\beta & 0 & -\lambda - \beta & \lambda \\
0 & \beta & 0 & -\lambda - \beta
\end{pmatrix}.
\]

This method is successful because of the following relations

\[ E[\mathbf{T}_{\mathbf{W}(0)\rightarrow (\text{absorbing})}] = \mathbf{W}(0) \int_0^\infty e^{\mathbf{A}t} dt, \tag{3} \]

and

\[ \int_0^\infty e^{\mathbf{A}t} dt = -\mathbf{A}^{-1}. \tag{4} \]

For configuration 1, the explicit expression for the \( \text{MTTF}_1 \) is given by
\[
E[T_{W(0)\rightarrow W\text{ (absorbing)}}] = \text{MTTF}_1 = \frac{2\lambda(\lambda + \alpha + \beta)^2 + \mu[(\lambda + \beta)^2 + \alpha(2\lambda + \beta)]}{\lambda^2[(\lambda + \alpha + \beta)^2 + \mu\alpha]}.
\] (5)

### 3.2. Calculations for Configuration 2

For configuration 2, the initial conditions are stated as follows:

**Initial conditions:**

The system has exactly 3 components are working initially and the service station is up, i.e., \( P_3(0) = 1 \), and all other initial probabilities are zero.

\[
P(0) = [P_3(0), P_2(0), P_1(0)] = [1, 0, 0], Q(0) = [Q_3(0), Q_2(0), Q_1(0)] = [0, 0, 0].
\]

Let \( W = [P, Q] \) denote the probability row vector. The differential equations written in matrix form are given by

\[
\dot{W} = WB_2,
\] (6)

where

\[
B_2 = \begin{pmatrix}
-2\lambda - \alpha & 2\lambda & 0 & \alpha & 0 & 0 \\
\mu & -2\lambda - \mu - \alpha & 2\lambda & 0 & \alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\beta & 0 & 0 & -2\lambda - \beta & 2\lambda & 0 \\
0 & \beta & 0 & 0 & -2\lambda - \beta & 2\lambda \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
\]

Without deriving the transient solutions, we still use the above procedure shown in configuration 1. We delete the third row and column, and the last row and column of matrix \( B_2 \) for the absorbing state \( (s) \) which yields a new matrix, say \( A \). The expected times to reach an absorbing state is calculated from

\[
E[T_{W(0)\rightarrow W\text{ (absorbing)}}] = W(0)(-A^{-1}) \begin{pmatrix}
1 \\
1 \\
1 \\
1
\end{pmatrix},
\] (7)

where \( W(0) = [P(0), Q(0)] = [1, 0, 0, 0] \), and

\[
A = \begin{pmatrix}
-2\lambda - \alpha & 2\lambda & \alpha & 0 \\
\mu & -2\lambda - \mu - \alpha & 0 & \alpha \\
\beta & 0 & -2\lambda - \beta & 2\lambda \\
0 & \beta & 0 & -2\lambda - \beta
\end{pmatrix}.
\]

For configuration 2, the explicit expression for the \( \text{MTTF}_2 \) is given by
\[
E[T_{W(0)\rightarrow W\ (\text{absorbing})}]=M^{TT}F_2 = \frac{4\lambda(2\lambda + \alpha + \beta)^2 + \mu[(2\lambda + \beta)^2 + \alpha(4\lambda + \beta)]}{4\lambda^2[(2\lambda + \alpha + \beta)^2 + \mu\alpha]}.
\] (8)

3.3. Calculations for Configuration 3

For configuration 3, the initial conditions are considered as follows:

**Initial conditions:**

The system has exactly 5 components are working initially and the service station is up, i.e., \( P_5(0) = 1 \), and all other initial probabilities are zero.

\[
P(0) = [P_5(0), P_4(0), P_3(0), P_2(0)] = [1, 0, 0, 0].
\]

\[
Q(0) = [Q_5(0), Q_4(0), Q_3(0), Q_2(0)] = [0, 0, 0, 0].
\]

Let \( W = [P, Q] \) denote the probability row vector. The differential equations written in matrix form are expressed as:

\[
W = WB_3,
\]

where

\[
B_3 = \begin{pmatrix}
-3\lambda - \alpha & 3\lambda & 0 & 0 & \alpha & 0 & 0 & 0 \\
\mu & -3\lambda - \mu - \alpha & 3\lambda & 0 & 0 & \alpha & 0 & 0 \\
0 & \mu & -3\lambda - \mu - \alpha & 3\lambda & 0 & 0 & \alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta & 0 & 0 & 0 & -3\lambda - \beta & 3\lambda & 0 & 0 \\
0 & \beta & 0 & 0 & 0 & -3\lambda - \beta & 3\lambda & 0 \\
0 & 0 & \beta & 0 & 0 & 0 & -3\lambda - \beta & 3\lambda \\
0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Again, we use the above procedure shown in configuration 1. We delete the fourth row and column, and the last row and column of matrix \( B_3 \) for the absorbing state (s) which gives a new matrix, say \( A \). The expected times to reach an absorbing state is evaluated from

\[
E[T_{W(0)\rightarrow W\ (\text{absorbing})}]=W(0)(-A^{-1}).
\]

(10)

where \( W(0) = [P(0), Q(0)] = [1,0,0,0,0,0] \), and
For configuration 3, the computer software, e.g., MATLAB, is used to obtain the explicit expression for the $MTTF_3$. The expression for the $MTTF_3$ is too ample to be shown here. However, a numerical example will given for the reliability case to compare the configuration 3 with other configurations.

4. Availability Analysis

4.1. Calculations for Configuration 1

Let $dP_1/dt = P_1$ and $dQ_1/dt = Q_1$. For the availability case of configuration 1, the differential-difference equations can be expressed as

$$
\begin{pmatrix}
P'_2 \\
P'_1 \\
P'_0 \\
Q'_2 \\
Q'_1 \\
Q'_0
\end{pmatrix} =
\begin{pmatrix}
-\lambda - \alpha & \mu & 0 & \beta & 0 & 0 \\
\lambda & -\lambda - \mu - \alpha & \mu & 0 & \beta & 0 \\
0 & \lambda & -\alpha - \mu & 0 & 0 & \beta \\
\alpha & 0 & 0 & -\lambda - \beta & 0 & 0 \\
0 & \alpha & 0 & \lambda & -\lambda - \beta & 0 \\
0 & 0 & \alpha & 0 & \lambda & -\beta
\end{pmatrix}
\begin{pmatrix}
P_2 \\
P_1 \\
P_0 \\
Q_2 \\
Q_1 \\
Q_0
\end{pmatrix}.
$$

Let $T_1$ denote the time-to-failure of the system for configuration 1. We use the following procedure to develop the steady-state availability. The steady-state availability is given by

$$A_{T_1}(\infty) = 1 - P_0(\infty) - Q_0(\infty).$$

In steady state, the derivatives of the state probabilities become zero, thus we obtain

$$
\begin{pmatrix}
-\lambda - \alpha & \mu & 0 & \beta & 0 & 0 \\
\lambda & -\lambda - \mu - \alpha & \mu & 0 & \beta & 0 \\
0 & \lambda & -\alpha - \mu & 0 & 0 & \beta \\
\alpha & 0 & 0 & -\lambda - \beta & 0 & 0 \\
0 & \alpha & 0 & \lambda & -\lambda - \beta & 0 \\
0 & 0 & \alpha & 0 & \lambda & -\beta
\end{pmatrix}
\begin{pmatrix}
P_2(\infty) \\
P_1(\infty) \\
P_0(\infty) \\
Q_2(\infty) \\
Q_1(\infty) \\
Q_0(\infty)
\end{pmatrix} =
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}.
$$

Using the following normalizing condition

$$\sum_{i=0}^3 P_i(\infty) + \sum_{i=0}^2 Q_i(\infty) = 1,$$

we substitute (14) in any one of the redundant rows in (13) to yield
Solving (15) to obtain the steady-state probabilities \( P_0(\infty) \) and \( Q_0(\infty) \) in the availability case. For configuration 1, the explicit expression for the \( A_{T_1}(\infty) \) is given by

\[
A_{T_1}(\infty) = \frac{\mu \beta \left( \lambda (\lambda + \alpha + \beta)^2 + \mu (\lambda + \beta)^2 + \alpha(2\lambda + \beta) \right)}{(\lambda + \beta)(\lambda + \alpha + \beta)^2 + \mu(\lambda + \mu)(\lambda + \beta)^2 + \lambda \mu \alpha (2\lambda + \beta)}.
\]

4.2. Calculations for Configuration 2

For the availability case of configuration 2, the differential-difference equations are given by

\[
\begin{pmatrix}
P_3 \\
P_2 \\
P_1 \\
Q_3 \\
Q_2 \\
Q_1
\end{pmatrix} =
\begin{pmatrix}
-2\lambda - \alpha & \mu & 0 & \beta & 0 & 0 \\
2\lambda & -2\lambda - \mu - \alpha & \mu & 0 & \beta & 0 \\
0 & 2\lambda & -\alpha - \mu & 0 & 0 & \beta \\
\alpha & 0 & 0 & -2\lambda - \beta & 0 & 0 \\
0 & \alpha & 0 & 2\lambda & -2\lambda - \beta & 0 \\
0 & 0 & \alpha & 0 & 2\lambda & -\beta
\end{pmatrix}
\begin{pmatrix}
P_3 \\
P_2 \\
P_1 \\
Q_3 \\
Q_2 \\
Q_1
\end{pmatrix}.
\]

Let \( T_2 \) represent the time-to-failure of the system for configuration 2. We calculate the steady-state availability with

\[
A_{T_2}(\infty) = 1 - P_1(\infty) - Q_1(\infty).
\]

In steady state, the derivatives of the state probabilities become zero, thus we obtain

\[
\begin{pmatrix}
-2\lambda - \alpha & \mu & 0 & \beta & 0 & 0 \\
2\lambda & -2\lambda - \mu - \alpha & \mu & 0 & \beta & 0 \\
0 & 2\lambda & -\alpha - \mu & 0 & 0 & \beta \\
\alpha & 0 & 0 & -2\lambda - \beta & 0 & 0 \\
0 & \alpha & 0 & 2\lambda & -2\lambda - \beta & 0 \\
0 & 0 & \alpha & 0 & 2\lambda & -\beta
\end{pmatrix}
\begin{pmatrix}
P_3 \\
P_2 \\
P_1 \\
Q_3 \\
Q_2 \\
Q_1
\end{pmatrix} =
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}.
\]

We have to satisfy the following normalizing condition

\[
\sum_{i=1}^{3} P_i(\infty) + \sum_{i=1}^{3} Q_i(\infty) = 1.
\]
Substituting this condition in any one of the redundant rows in (18) gives

\[
\begin{pmatrix}
-2\lambda - \alpha & \mu & 0 & \beta & 0 & 0 & 0 \\
2\lambda & -2\lambda - \mu - \alpha & \mu & 0 & \beta & 0 & \mathbf{P}_3(\infty) \\
0 & 2\lambda & -\alpha - \mu & 0 & 0 & \beta & \mathbf{P}_2(\infty) \\
\alpha & 0 & 0 & -2\lambda - \beta & 0 & 0 & \mathbf{P}_1(\infty) \\
1 & 1 & 1 & 1 & 1 & 1 & \mathbf{Q}_3(\infty) \\
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}.
\] (19)

The solutions of (19) provides \( \mathbf{P}_1(\infty) \) and \( \mathbf{Q}_1(\infty) \). For configuration 2, we obtain the explicit expression for the \( A_{T_2}(\infty) \)

\[
A_{T_2}(\infty) = \frac{\mu \beta \{2\lambda (2\lambda + \alpha + \beta)^2 + \mu [(2\lambda + \beta)^2 + \alpha (4\lambda + \beta)]\}}{(\alpha + \beta) [4\lambda^2 (2\lambda + \alpha + \beta)^2 + \mu (2\lambda + \mu)(2\lambda + \beta)^2 + 2\lambda \mu \alpha (4\lambda + \beta)]}.
\] (20)

**4.3. Calculations for Configuration 3**

For the availability case of configuration 3, the steady-state equations are given by

\[
\begin{pmatrix}
-3\lambda - \alpha & \mu & 0 & 0 & \beta & 0 & 0 & 0 \\
3\lambda & -3\lambda - \mu - \alpha & \mu & 0 & 0 & \beta & 0 & 0 \\
0 & 3\lambda & -3\lambda - \mu - \alpha & \mu & 0 & 0 & \beta & 0 \\
0 & 0 & 3\lambda & -\mu - \alpha & 0 & 0 & 0 & \beta \\
\alpha & 0 & 0 & 0 & -3\lambda - \beta & 0 & 0 & 0 \\
0 & \alpha & 0 & 0 & 3\lambda & -3\lambda - \beta & 0 & 0 \\
0 & 0 & \alpha & 0 & 0 & 3\lambda & -3\lambda - \beta & 0 \\
0 & 0 & 0 & \alpha & 0 & 0 & 3\lambda & -\beta \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\mathbf{P}_3(\infty) \\
\mathbf{P}_4(\infty) \\
\mathbf{P}_3(\infty) \\
\mathbf{P}_2(\infty) \\
\mathbf{Q}_3(\infty) \\
\mathbf{Q}_4(\infty) \\
\mathbf{Q}_3(\infty) \\
\mathbf{Q}_2(\infty) \\
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}.
\] (21)

Let \( T_3 \) represent the time-to-failure of the system for configuration 3. The steady-state availability for configuration 3 is

\[
A_{T_3}(\infty) = 1 - P_2(\infty) - Q_2(\infty).
\] (22)
Solving (21) and using the following normalizing conditions
\[
\sum_{i=2}^{5} p_i(\infty) + \sum_{i=2}^{5} q_i(\infty) = 1,
\]  
we obtain \( p_2(\infty) \) and \( q_2(\infty) \). For configuration 3, the computer software, e.g., MATLAB, is used to develop the explicit expression for the \( A_{T_i}(\infty) \). The expression for the \( A_{T_i}(\infty) \) is too spacious to be shown here. However, a numerical example will given for the availability case to compare the configuration 3 with other configurations.

5. Comparison of the Three Configurations

The purpose of this section is to present specific numerical comparisons for the mean time to system failure, \( MTTF \), and the steady-state availability, \( A_T(\infty) \). The computer software, e.g., MATLAB, is used to compare three configurations in terms of their \( MTTF_i \) and \( A_T(\infty) \), where \( i = 1, 2, 3 \).

5.1. Comparison for the \( MTTF_i \)

We perform a comparison for the \( MTTF \) of the configurations 1, 2, and 3. The following numerical results are obtained by considering the following system parameters:

Case 1: We fix \( \mu = 1.0, \alpha = 0.1, \beta = 3.0 \), and vary the values of \( \lambda \) from 0.1 to 1.5.
Case 2: We fix \( \lambda = 0.3, \alpha = 0.1, \beta = 3.0 \), and vary the values of \( \mu \) from 0.5 to 2.0.
Case 3: We fix \( \lambda = 0.3, \mu = 1.0, \beta = 3.0 \), and vary the values of \( \alpha \) from 0.05 to 0.4.
Case 4: We fix \( \lambda = 0.3, \mu = 1.0, \alpha = 0.1 \), and vary the values of \( \beta \) from 1.0 to 9.0.

Numerical results of the \( MTTF \) for configuration \( i(i = 1, 2, 3) \) are shown in Table 3 for cases 1-4, respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of parameter</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1 &lt; ( \lambda ) &lt; 1.3253, 1.3253 &lt; ( \lambda ) &lt; 1.5</td>
<td>( MTTF_1 &gt; MTTF_3 &gt; MTTF_2 ), ( MTTF_1 &gt; MTTF_2 &gt; MTTF_3 )</td>
</tr>
<tr>
<td>2</td>
<td>0.5 &lt; ( \mu ) &lt; 2.0</td>
<td>( MTTF_1 &gt; MTTF_3 &gt; MTTF_2 )</td>
</tr>
<tr>
<td>3</td>
<td>0.05 &lt; ( \alpha ) &lt; 0.4</td>
<td>( MTTF_1 &gt; MTTF_3 &gt; MTTF_2 )</td>
</tr>
<tr>
<td>4</td>
<td>1.0 &lt; ( \beta ) &lt; 9.0</td>
<td>( MTTF_1 &gt; MTTF_3 &gt; MTTF_2 )</td>
</tr>
</tbody>
</table>

Table 4. Comparison of the configurations 1, 2, 3 for \( A_T(\infty) \).

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of parameter</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1 &lt; ( \lambda ) &lt; 0.2054, 0.2054 &lt; ( \lambda ) &lt; 1.5</td>
<td>( A_{T_1}(\infty) &gt; A_{T_2}(\infty) &gt; A_{T_3}(\infty) ), ( A_{T_1}(\infty) &gt; A_{T_3}(\infty) &gt; A_{T_2}(\infty) )</td>
</tr>
<tr>
<td>2</td>
<td>0.5 &lt; ( \mu ) &lt; 1.4609, 1.4609 &lt; ( \mu ) &lt; 2.0</td>
<td>( A_{T_1}(\infty) &gt; A_{T_2}(\infty) &gt; A_{T_3}(\infty) ), ( A_{T_1}(\infty) &gt; A_{T_3}(\infty) &gt; A_{T_2}(\infty) )</td>
</tr>
<tr>
<td>3</td>
<td>0.05 &lt; ( \alpha ) &lt; 0.4</td>
<td>( A_{T_1}(\infty) &gt; A_{T_2}(\infty) &gt; A_{T_3}(\infty) )</td>
</tr>
<tr>
<td>4</td>
<td>1.0 &lt; ( \beta ) &lt; 9.0</td>
<td>( A_{T_1}(\infty) &gt; A_{T_2}(\infty) &gt; A_{T_3}(\infty) )</td>
</tr>
</tbody>
</table>
5.2. Comparison for the $A_T(\infty)$

We perform a comparison for the $A_T(\infty)$ of the configurations 1, 2, and 3. Numerical results of the $A_T(\infty)$ for configuration $i (i = 1, 2, 3)$ are shown in Table 4 for cases 1-4, respectively.

5.3. Comparison of Three Configurations Based on Their Cost/Benefit Ratios

We consider that the various configurations may have different costs when comparing all configurations. From Table 2, the costs $C_i$ of the configuration $i (i = 1, 2, 3)$ are listed in the following:

\[
C_1 = 48 \times 10^6, C_2 = 39 \times 10^6, C_3 = 42 \times 10^6.
\]

Numerical results of the $C_i/M\text{MTTF}_i$ for configuration $i (i = 1, 2, 3)$ are depicted in Figures 4-7 for cases 1-4, respectively. Figures 4 and 6 show that the $C_i/M\text{MTTF}_i$ increases as $\lambda$ or $\alpha$ increases for any configuration. On the other hand, Figures 5 and 7 show that the $C_i/M\text{MTTF}_i$ decreases as $\mu$ or $\beta$ increases for any configuration. We can easily observe from Figures 4-7 that the optimal configuration using the $C_i/M\text{MTTF}_i$ value is configuration 1.

![Figure 4. $C_i/M\text{MTTF}_i$ versus failure rate $\lambda$.](image)

![Figure 5. $C_i/M\text{MTTF}_i$ versus service rate $\mu$.](image)
Cost Benefit Analysis of Series Systems

Figure 6. $C_i / \text{MTTF}_i$ versus breakdown rate $\alpha$.

Figure 7. $C_i / \text{MTTF}_i$ versus repair rate $\beta$.

Numerical results of the $C_i/A_T(\infty)$ for configuration $i(i=1,2,3)$ are depicted in Figures 8-11 for cases 1-4, respectively. Figures 8 and 10 show that the $C_i/A_T(\infty)$ increases as $\lambda$ or $\alpha$ increases for any configuration. On the other hand, Figures 9 and 11 show that the $C_i/A_T(\infty)$ decreases as $\mu$ or $\beta$ increases for any configuration. One observes from Figure 8 that the optimal configuration using the $C_i/A_T(\infty)$ value depends on the value of $\lambda$. When $0.1 < \lambda < 0.4047$, the optimal configuration is configuration 2, but when $0.4047 < \lambda < 1.5$, the optimal configuration is configuration 1. One can easily see from Figure 9 that the optimal configuration using the $C_i/A_T(\infty)$ value depends on the value of $\mu$. When $0.5 < \mu < 0.7405$, the optimal configuration is configuration 1, but when $0.7405 < \mu < 2.0$, the optimal configuration is configuration 2. It appears from Figures 10-11 that the optimal configuration using the $C_i/A_T(\infty)$ value does not depend on the value of $\alpha$ and $\beta$, respectively. We see from Figures 10-11 that the optimal configuration using the $C_i/A_T(\infty)$ value is configuration 2.
Figure 8. $C_i / \frac{A_T}{(\infty)}$ versus failure rate $\lambda$.

Figure 9. $C_i / \frac{A_T}{(\infty)}$ versus service rate $\mu$.

Figure 10. $C_i / \frac{A_T}{(\infty)}$ versus breakdown rate $\alpha$. 

\[
\frac{C_i}{A_T(\infty)} \times 10^8
\]

- **Configuration 1**
- **Configuration 2**
- **Configuration 3**

$\lambda = 0.4047$, $\mu = 0.7405$, $\alpha = 0.1$, $\beta = 3.0$

$\lambda = 0.2548$, $\mu = 1.1782$, $\alpha = 0.1$, $\beta = 3.0$

$\mu = 1.0$, $\alpha = 0.1$, $\beta = 3.0$
6. Conclusions

In this paper, we constructed three different series system configurations with cold standby components and a repairable service station to study the cost/benefit analysis of three configurations under uncertainty. We have developed the explicit expressions for the $MTTF$ and the $A_r(\infty)$ for three configurations and performed a comparative analysis. We rank three configurations based on the $MTTF$, the $A_r(\infty)$, and the $C/B$, where $B$ is either $MTTF$ or $A_r(\infty)$. It is interesting to mention first that the optimal configuration using the cost/MTTF measure is configuration 1. Next, using the cost/$A_r(\infty)$ measure, the optimal configuration depends on the values of $\lambda$ and $\mu$ but does not depend on the values of $\alpha$ and $\beta$.

References


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