Threshold \( N \)-Policy for Degraded Machining System with Multiple Type Spares and Multiple Vacations

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Abstract: This paper deals with a multi-component machining system consisting of \( M \) operating units along with \( k \) types of spare machines. The repair facility comprises of \( R \) heterogeneous repairmen who get switched on one-by-one when the number of failed units reaches a certain pre-determined level \( N_i \) for \( i^{th} \) repairman; \( i = 1, 2, \ldots, R \). The repairman can leave for a vacation of random length during its idle time. The \( i^{th} \) repairman would leave for vacation if the number of failed units drops below \( N_i \). The first repair facility, however would leave for vacation only when the system is empty. Once all the spares are utilized, the system starts working in degraded mode due to load sharing. The repairmen may breakdown individually or due to common cause according to the Poisson process. The mean failure and repair rates of the machines depend on the status of the system. The matrix recursive method is used to find the steady-state probabilities of the number of failed machines in the system as well as other performance measures. The model under consideration is computationally tractable which is illustrated by means of numerical experiments. We also approximate the performance measures by using a special class of neuro-fuzzy systems, i.e. Adaptive Network-based Fuzzy Interference Systems (ANFIS) which can identify parameters by using supervised learning methods. Some special cases have also been deduced to tally our results in particular cases with the existing results.

Keywords: Common cause failure, degraded failure, heterogeneous servers, machine repair, matrix recursive method, mixed spares, multiple vacations, threshold policy.

1. Introduction

The rapid and spectacular developments and progress of the machining system definitely indicates its tremendous potentialities in promoting and sustaining industrial as well as economic growth and stability. Machines are an integral part of any production/manufacturing system as such the importance of performance modeling of the machining system cannot be ignored. Spare part support is most important factor for the smooth running of any machining system. The provision of spares may be helpful in improving the system reliability/availability by introducing redundancy as additional equipments, assemblies, devices, etc.. The machine repair problem with spares has been an area of interest for many researchers since long. Instead of going to the details of such earlier works, it is worthwhile to give a brief overview of some important contributions in recent past. The machine interference problem with standbys has been investigated by several researchers including Al-seedy [1], Gupta and Rao [10], Jain [12], etc. Shawky [32] has developed a machine interference model \( M/M/c/k/N \) with spares and visualized its effects under balking and reneging behaviour of the failed units. Jain et al. [20] examined some significant effects of the balking behaviour of the failed units on the machining system having multi-repairmen. A repairable machining system with the provision of spares has also been studied by Jain and Baghel [14]. It is sometimes beneficial to use mixed standbys...
instead of a single type due to some techno-economic constraints. This feature can be seen in the work of Sharma et al. [31] who have done the performance modeling of multi component machining system having mixed standby components. The reliability analysis of $k$-out-of-$N$ machining system with spares has been studied by Jain et al. [19]. Recently, the profit analysis of $M/M/R$ machine repair problem is done by Wang et al. [38] who have incorporated the concepts of balking, reneging and standby switching failure.

The machine repair problems or queueing problems with server vacations have been investigated by many researchers with several combinations. The multi-server queueing problems with vacation were examined by Kao and Narayanan [23], Tian et al. [36], Kumar and Madheswari [26], Tian and Zhang [35], etc. The bilevel control of degraded machining system with warm standbys was studied by Jain et al. [18]. They have also incorporated setup times and vacations taken by the server. Arumuganathan and Jeyakumar [2] did the steady state analysis of a bulk queue in which the server takes setup time, closedown time and vacations and can serve the customers according to $N$-policy. Xu and Zhang [39] developed a single vacation $(e, d)$ policy for markovian multi-server queue. Recently, the vacation policies for machine repair problem consisting of $M$ operating units with two type spare machines and multi repairmen were examined by Ke and Wang [25].

When there are less failed units than the number of repairmen in a machining system, the repairmen may be idle. This results in the waste of valuable resources and is uneconomical. A threshold policy to turn on the server based on the queue size can play a significant role to resolve this issue. Such policy in case of single server queue is termed as $N$-policy in which the server will not initiate the service until queue size reaches a pre assigned level. A few researchers have applied this policy for many queueing systems of different variants and their findings are reported in the literature too. Kavusturucu and Gupta [24] analyzed the finite buffer tandem manufacturing systems with $N$-policy. The finite source queueing system with warm spares under $N$-policy has been examined by Gupta [9]. The recent work in this field has been done by Kumar et al. [27], Jain [13], Jain et al. [17], etc.

The common cause failures have a major impact on the availability and reliability of the redundant repairable systems. Several environment conditions such as variation in temperature, humidity, vibration, or shock voltage fluctuation, etc. that prevail in many applications can cause simultaneous failure of some or all units of the system. Some researchers who have incorporated this concept in their works include Hughes [11], Kvam and Miller [28], Jain et al. [16], Vaurio [37], Jain and Mishra [15], Dai et al. [5], etc.

Neuro-fuzzy technique is an emerging soft computing technique which has been successfully applied in telecommunication systems, electronic goods, automobiles with automatic transmission, financial engineering, etc. Fuzzy technology for studying neural networks was used by Takagi [34]. Later on, Cox [4] integrated fuzzy logic into neural networks. Jang and Sun [21, 22] developed neuro-fuzzy model and controller. Recently, Diab [6], Ciaramella et al. [3] and Shao et al. [30] worked on Neuro-fuzzy systems.

In this paper, we present investigation on multi component machining system with a repair crew and spare part support apart from operating machines. This study is the generalization of the work done by Ke and Wang’s [25] $M/M/R$ machine repair model with vacation and two types of spares by incorporating additional features namely (i) multiple type spares, (ii) heterogeneous servers, (iii) common cause failure, (iv) degraded failure and (v) threshold $N$-policy. We formulate various performance measures of the machining system in which the repairmen turn on according to a pre-specified policy and can go for multiple
vacations. The organization of the rest of the paper is as follows. The mathematical formulation of the model by stating the requisite assumptions is done in section 2. The steady-state probabilities and the governing equations is facilitated in section 3. Next, we employ the matrix recursive method for the solution purpose in section 4. Further, some performance characteristics are established in section 5. Section 6 is devoted to deduce some special cases of the model under consideration. The numerical illustration and sensitivity analysis have been put forward in sections 7 and 8 respectively. Finally, the paper ends with the insights of future prospects and concluding remarks in section 9.

2. System Description and Assumptions

Multi component machining systems with spares find significant applications in manufacturing and production industries, which are devoted to producing high-quality products in the most economical and timely manner. Queueing indices not only indicate the customer satisfaction level, but also measure the performance of a machining system in order to improve and upgrade it in future design and development phases under techno-economic constraints. Here, we are going to develop queueing model of multi-component system with the provision of spare part support. The machining system consists of $L$ machines, out of which $M$ identical machines operate simultaneously in parallel and the remaining $S = L - M$ machines are provided as warm standby units which are of $k$ types; $S_j$ ($j = 1, 2, ..., k$) warm standby machines are of $j^{th}$ type; $S = \sum_{j=1}^{k} S_j$. The operating machines and the $j^{th}$ ($j = 1, 2, ..., k$) type spare machines are subjected to breakdown in Poisson fashion with rates $\lambda_j$ and $\alpha_j$ ($j = 1, 2, ..., k$), respectively. It is assumed that the $j^{th}$ type spare have higher failure rate than the $(j-1)^{th}$ ($j = 1, 2, ..., k$) type spare. When an operating machine fails, it will immediately be replaced by an available spare which has higher failure rate. When a type $j$ ($j = 1, 2, ..., k$) spare machine replaces the failed operating machine, its failure characteristics will be that of an operating machine.

There is a facility of $R$ heterogeneous repairmen; the repair time of the $i^{th}$ ($i = 1, 2, ..., R$) repairman is assumed to be exponential distributed with parameter $\mu_i$ ($i = 1, 2, ..., R$) and the repair time of all repairmen are assumed to be independent. We denote $\mu^{(i)} = \sum_{j=1}^{k} \mu_j$. Here $l$ ($l = 1, 2, ..., R$) denotes the threshold level at which $i^{th}$ repairman returns from vacation. The mean repair rate $\mu_n$ is given by

$$\mu_n = \begin{cases} 
\mu^{(i)}; & l < n \leq L; 1 \leq l \leq R \\
\mu_i; & n = l; 1 \leq l \leq R \\
0; & \text{otherwise,}
\end{cases}$$

where, $\mu^{(i)} = \sum_{j=1}^{k} \mu_j; 1 \leq l \leq R$.

When a failed machine is repaired, it is as good as before failure and joins the standby group of machines unless the system is short, in that case it is put into operation along with operating machines.

Under the threshold policy considered, the $i^{th}$ ($i = 1, 2, ..., R$) repairman turns on when the number of failed units reaches the threshold value $N_i$ ($i = 1, 2, ..., R$); during this period $S^{(i)} = \sum_{j=1}^{i} S_j$ spares are used. The first repairman once switched on, would leave for vacation only when the system becomes empty. The $i^{th}$ ($i = 1, 2, ..., R$) repairman would however leave for vacation as soon as the number of failed units drops below the threshold value $N_i$ ($i = 1, 2, ..., R$). The vacation times of repairmen are assumed to be exponentially distributed with rate $\nu$. 
When all the spare machines are being used up and a new machine fails, the system
works under stress in degraded mode with degraded failure rate \( \lambda_d \). The operating machines
may also fail due to common cause according to the Poisson process with rate \( \lambda_c \). The
switchover time from repair to operating or standby group is negligible.

The mean failure rate \( \lambda_n \) is given by

\[
\lambda_n = \begin{cases} 
M \lambda + (S^{(i)} - n) \alpha_i + \sum_{i=2}^{k} S_i \alpha_i + \lambda_c \lambda_d ; & 0 \leq n < S^{(i)}; \\
M \lambda + (S^{(i)} - n) \alpha_j + \sum_{i=j+1}^{k} S_i \alpha_i + \lambda_c \lambda_d ; & S^{(i)} \leq n < S^{(i+1)}; \\
(L - n) \lambda_d ; & n \leq M + S^{(i)} = L \\
0; & \text{otherwise.}
\end{cases}
\]

3. Steady-State Probabilities

The state of the system is described by the pairs \( \{(i, n); i=1, 2, \ldots, R \text{ and } n = i, i+1, \ldots, L\} \) where \( i \) and \( n \) denote the number of busy repairmen and the failed machines in
the system, respectively. We define the steady state probabilities as follows:

\( P_{0,n} \): Probability that there are \( n \) failed machines in the system when all repairmen are on vacation.

\( P_{i,n} \): Probability that there are \( n \) failed machines in the system when there are \( i \) repairmen working and remaining \( R - i \) are on vacation.

\( P_{R,n} \): Probability that there are \( n \) failed machines in the system when all the repairmen are working.

By using appropriate transition rates as shown in Figure. 1, we construct Chapmann
Kolmogorov equations governing the model as follows:

(i) For \( l=0 \)

\[
\lambda_0 P_{0,0} = \mu_i P_{1,1},
\]

\[
\lambda_n P_{0,n} = \lambda_{n-1} P_{0,n-1}; 1 \leq n \leq N_1,
\]

\[
(\lambda_n + R_v) P_{0,n} = \lambda_{n-1} P_{0,n-1}; N_1 \leq n \leq L-1,
\]

\[
R_v P_{0,L} = \lambda_{L-1} P_{0,L-1}.
\]

(ii) For \( l=1, 2, \ldots, R-1 \)

\[
(\lambda_l + \mu_i) P_{i,l} = \mu_i P_{i+1,l+1} + \mu_{i+1} P_{i+1,l+1}; n = l,
\]

\[
(\lambda_i + \mu_i) P_{i,n} = \mu_i P_{i+1,n+1} + \lambda_{n-1} P_{i,n-1}; l < n < N_i,
\]

\[
(\lambda_n + \mu_i) P_{i,n} = \mu_i P_{i+1,n+1} + \lambda_{n-1} P_{i,n-1} + (R - l + 1) \nu P_{l-1,n}; N_i \leq n < N_{i+1},
\]

\[
(\lambda_n + \mu_i) (R - l) v P_{i,n} = \mu_i P_{i+1,n+1} + \lambda_{n-1} P_{i,n-1} + (R - l + 1) v P_{l-1,n}; N_R \leq n \leq L - 1,
\]

\[
(\mu_i + (R - l) v) P_{i,L} = \lambda_{L-1} P_{i+1,L-1} + (R - l + 1) v P_{l-1,L}.
\]
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(iii) For \( l = R \)

\[
\begin{align*}
(\lambda_n + \mu^{(i)}_n)P_{i,n} &= \lambda_{n+1}P_{i,n+1} + \mu^{(i)}_{i+1}P_{i,n+1}^3; l < n < N_i, \\
(\lambda_n + \mu^{(i)}_n)P_{i,n} &= \lambda_{n+1}P_{i,n+1} + \mu^{(i)}_{i+1}P_{i,n+1}^1 + (R-l+1)vP_{i,1}; N_i < n < L-1, \\
\mu^{(i)}_nP_{i,L} &= \lambda_{L+1}P_{i,L-1} + (R-l+1)vP_{i,1}.
\end{align*}
\]

Figure 1. State-transition rate diagram for the \( M/\mu M/R \) machine repair problem with three types of spares and multiple vacations (for \( R = 3, L = 20, N_1 = 2, N_2 = 6 \) and \( N_3 = 10 \)).

4. Matrix Recursive Method

Due to today's dynamic market, manufacturers require machining system that provides low cost, high quality and highly responsive solutions. These manufacturing systems can be designed in many different configurations. Hence it is important to analyze a system's performance quickly and effectively. A systematic method for evaluating the overall performance of embedded machining system configurations is needed. For studying the transient as well as steady state behaviour of any system, the matrix recursive method is applied. The benefit of using matrix technique for this model is that it avoids the need to solve steady-state equations directly and instead replaces them with matrices, which can be solved using well-known numerical techniques. The program for the same can be developed in any mathematical software such as MATLAB, Mathcad, Mathematica, Maple, etc.
We have employed matrix recursive method to obtain the probabilities of the system states and other performance measures. It is worthwhile to discuss the mathematical analysis based on matrix recursive approach employed for solution purpose in brief as follows:

4.1. Transition Probability Matrix

Let us consider an irreducible Markov chain with transition probability matrix \( \tilde{Q} \), which can be represented, in the following block tri-diagonal structure given by

\[
\tilde{Q} = \begin{bmatrix}
A_0 & B_0 & 0 & \cdots & \cdots & 0 \\
C_1 & A_1 & B_1 & 0 & \cdots & 0 \\
0 & C_2 & A_2 & B_2 & 0 & \cdots \\
\vdots & 0 & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & 0 & C_{R-2} & A_{R-2} & B_{R-2} & 0 \\
\vdots & \vdots & \vdots & 0 & C_{R-1} & A_{R-1} & B_{R-1} \\
0 & 0 & 0 & 0 & 0 & C_R & A_R
\end{bmatrix},
\]

where \( A_i, B_i \) and \( C_i \) (\( i = 0, 1, 2, \ldots, R \)) are block sub matrices. These matrices are given by

\[
A_i = \begin{bmatrix}
A_{i1} \\
A_{i2}
\end{bmatrix} \quad \text{(for } 0 \leq i \leq R-1), \quad B_i = \begin{bmatrix}
B_{i1} & B_{i2} \\
B_{i3} & B_{i4}
\end{bmatrix} \quad \text{(for } 0 \leq i \leq R-1),
\]

\[
C_i = \begin{bmatrix}
C_{i1} & C_{i2}
\end{bmatrix} \quad \text{(for } 1 \leq i \leq R), \quad C_{i1} = \begin{bmatrix}
\mu_i & 0 & \cdots & 0
\end{bmatrix}, \quad B_{4i} = \text{diag}[(R-i)v],
\]

\[
A_{i1} = \begin{bmatrix}
-(\lambda_i + \mu_i) & \lambda_i & 0 & 0 & \cdots & 0 \\
\mu_i & -(\lambda_{i+1} + \mu_i) & \lambda_{i+1} & \cdots & \cdots & \cdots \\
\cdots & \cdots & \ddots & \ddots & \ddots & \ddots \\
\mu_i & -(\lambda_{N_i} + \mu_i) & \lambda_{N_i} & 0 & \cdots & 0
\end{bmatrix},
\]

\[
A_{i2} = \begin{bmatrix}
0 & 0 & -\lambda_{N_i} + \mu_i + (R-i)v & \lambda_{N_i} & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \mu_i & -\lambda_{L-1} + \mu_i + (R-i)v & \lambda_{L-1}
\end{bmatrix},
\]

and

\[
A_R = \begin{bmatrix}
-\{\lambda_R + \mu_R\} & \lambda_R & 0 & 0 & \cdots & 0 \\
\mu_R & -\{\lambda_{R+1} + \mu_R\} & \lambda_{R+1} & 0 & \cdots & 0 \\
0 & \mu_R & -\{\lambda_{R+2} + \mu_R\} & \lambda_{R+2} & \cdots & 0 \\
0 & \cdots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 0 & \mu_R & -\{\lambda_L + \mu_R\}
\end{bmatrix}.
\]
Where $A_i, B_i$ and $C_i$ are the matrices of order $(L-i+1) \times (L-i+1), (L-i+1) \times (L-i)$ and $(L-i) \times (L-i+1)$, respectively. $B_{ii}, B_{ij}$ and $B_{ij}$ are zero matrices of order $(N_{i+1}-i) \times (N_{i+1}-i-1), (N_{i+1}-i) \times (N_{i+1}+1)$ and $(L-N_{i+1}+1) \times (N_{i+1}-i-1)$, respectively. The order of $B_{ii}$ is $(L-N_{i+1}+1) \times (L-N_{i+1}+1)$ and the order of matrices $A_{ii}$ and $A_{ij}$ are $(N_{i+1}-i) \times (L-i+1)$ and $(L-N_{i+1}+1) \times (L-i+1)$, respectively.

4.2. Computation of Probabilities

Let the steady-state probability vector of $\tilde{Q}$ be denoted by $P$ which can be partitioned as $P = \{P_0, P_1, P_2, \ldots, P_R\}$ where $P_i = \{P_{i,0}, P_{i,1}, P_{i,2}, \ldots, P_{i,L}\}$, for $0 \leq i \leq R$, is a $1 \times (L+1-i)$ vector.

This vector $P$ can be obtained by using $P \tilde{Q} = 0$ with the normalizing condition

$$
\sum_{i=0}^{R} P_i e = 1, \quad (14)
$$

where $e$ is a column vector with each component equal to one.

The set of steady-state equations $P \tilde{Q} = 0$ are given by

$$
P_0 A_0 + P_1 C_1 = 0, \quad (15)
$$

$$
P_{i+1} B_{i+1} + P_i A_i + P_{i+1} C_{i+1} = 0; \quad 1 \leq i \leq R-1, \quad (16)
$$

$$
P_{R-1} B_{R-1} + P_R A_R = 0. \quad (17)
$$

Equation (17) yields

$$
P_R = -P_{R-1} B_{R-1} A_R^{-1}. \quad (18)
$$

The value of steady-state probability vector $P_i$ is a function of the transition rates between states with $i-1$ queued customers and states with $i$ queued customers. Since these transition rates don’t depend upon $i$, there is a constant matrix $X_i$ such that

$$
P_i = P_{i-1} X_i \quad \text{for} \quad 1 \leq i \leq R-1. \quad (19)
$$

Substituting equation (19) for $i = 1$ in equation (15), we get

$$
P_0 (A_0 + X_1 C_1) = 0, \quad (20)
$$

where

$$
X_i = -B_{i-1} (A_i + X_{i+1} C_{i+1})^{-1}, \quad \text{for} \quad i = 1, 2, \ldots, R-1, \quad (21)
$$

is a $(L+2-i) \times (L+1-i)$ matrix.

We also have,

$$
X_R = -B_{R-1} A_R^{-1}. \quad (22)
$$

We can determine $P_0$ by using equation (20). The other probabilities $P_{R}, P_{R-1}, \ldots, P_1$ can be determined by using equations (19), (21) and (22) such that these probabilities satisfy the normalizing condition given in equation (14).
5. Performance Measures

To evaluate the overall performance of the machining system under consideration, we establish some queueing characteristics which are as follows:

- The expected number of failed machines in the system is given by
  \[ E[N] = \sum_{i=0}^{R} \sum_{n=1}^{L} n P_{i,n}. \]  
  (23)

- The expected number of failed machines in the queue is
  \[ E[N_q] = \sum_{i=0}^{R} \sum_{n=1}^{L} (n-i) P_{i,n}. \]  
  (24)

- The expected number of operating machines in the system is
  \[ E[O] = M - \sum_{i=0}^{R} \sum_{n=1}^{L} (n-i) P_{i,n}. \]  
  (25)

- The expected number of spare machines in the system acting as standbys is
  \[ E(S) = \sum_{i=0}^{R} \sum_{n=1}^{S} (S-n) P_{i,n}. \]  
  (26)

- The expected number of spare machines of type \( j \) \((j=1,2,\ldots,k-1)\) in the system acting as standbys is obtained using
  \[ E(S_j) = \sum_{i=0}^{R} \sum_{n=1}^{S_j} (S_j-n) P_{i,n}. \]  
  (27)

- The expected number of busy servers in the system is given by
  \[ E[B] = \sum_{i=1}^{R} \sum_{n=1}^{L} i P_{i,n}. \]  
  (28)

- The expected number of vacationing servers in the system is given by
  \[ E[V] = \sum_{i=0}^{R} \sum_{n=1}^{L} i P_{i,n}. \]  
  (29)

- The fraction of total time that the machines are working (i.e. machine availability) is
  \[ M.A. = I - \frac{E[N]}{L}. \]  
  (30)

- The effective arrival rate into the system is
  \[ \lambda_{\text{eff}} = \sum_{i=0}^{R} \sum_{n=1}^{L} \lambda_n P_{i,n}. \]  
  (31)

- The expected waiting time in the system and in the queue are
  \[ E[W] = \frac{E[N]}{\lambda_{\text{eff}}} \quad \text{and} \quad E[W_q] = \frac{E[N_q]}{\lambda_{\text{eff}}}. \]  
  (32)
6. Special Cases

We have deduced some special cases by setting appropriate parameter values to the respective formulae. The service rates for different cases are as follows:

Case I: $M/M/R$ machine repair problem with warm spares

In this case, we set $\nu = 0$, $k = 2$, $\alpha_1 = \alpha_2$, $\lambda_c = 0$, $\lambda_d = \lambda$, and

$$\mu_n = \begin{cases} 
i \mu; n = i, i + 1, \ldots, R \\
R \mu_1; n = R, R + 1, \ldots, M + S = L \\
0; otherwise. \end{cases}$$

In this case, our model reduces to that of Sivazlian and Wang [33].

Case II: $M/M/1$ machine repair problem with spare and server multiple vacations (or single vacation)

In this case, we put $R = 1$, $k = 2$, $\alpha_1 = \alpha_2$, $\lambda_c = 0$, $\lambda_d = \lambda$, and

$$\mu_n = \begin{cases} 
i \mu; n = i, i + 1, \ldots, R \\
R \mu_1; n = R, R + 1, \ldots, M + S = L \\
0; otherwise. \end{cases}$$

For this special case, our model coincides with that of Gupta [8].

Case III: $M/M/R/L$ queueing system

In this case, we set $\nu = 0$, $\alpha_1 = \alpha_2 = \cdots = \alpha_k = 0$, $S = L - 1$, $\lambda_c = 0$, $\lambda_d = \lambda$, and

$$\mu_n = \begin{cases} 
i \mu; n = i, i + 1, \ldots, R \\
R \mu_1; n = R, R + 1, \ldots, M + S = L \\
0; otherwise. \end{cases}$$

Then our model matches with that of Gross and Harris [7].

Case IV: $M/M/R$ machine repair problem with two type spares and server multiple vacations (or single vacation)

In this case, we put $k = 2$, $\lambda_c = 0$, $\lambda_d = \lambda$, and

$$\mu_n = \begin{cases} 
i \mu; n = i, i + 1, \ldots, R \\
R \mu_1; n = R, R + 1, \ldots, M + S = L \\
0; otherwise. \end{cases}$$

The model for this particular case coincides with the recently developed model by Ke and Wang [25].

7. Numerical Illustration

In this section, we discuss the numerical tractability of our model for evaluating the stationary distribution of the system size and other performance measures by taking an illustration. Consider a dry-cleaning store which has the facility of ten machines. The operating characteristics of these machines are such that any machine breakdowns according
to Poisson process with mean breakdown rate $\lambda$. In order to ensure a continuous service, the store manager has an option to replace the failed machines by three types of spare machines. There are 2 spare machines of type 1 whereas 4 spare machines each of type 2 and type 3. The failure rate of $i^{th}$ ($i = 1, 2, 3$) type machine is $\alpha_i$ such that $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \lambda$.

The store manager provides three heterogeneous caretakers (i.e. repairmen) who are in charge of monitoring, maintaining and repairing these failed machines (either operating or spare). The repair (service) times of failed machines provided by these workers are assumed to be exponential distributed depending upon the number of failed machines in the store. When these repairmen are not operating, they are assigned some secondary jobs like cleaning the store, taking care of dry-cleaned clothes, etc. which correspond to the vacation of the repairmen. The store manager follows a threshold policy to repair the failed machines. According to this policy, initially, on the failure of two machines, the first repairman is called upon from the vacation. If another six machines fail, the second repairman returns from the vacation and if next four machines fail, the third repairman is switched on after coming from vacation. The repairmen continue repairing the failed machines until no failed machine is left in the store. There may be the case that all the three types of spare machines are being used up and a new machine fails. At that time, the system describes above works under degraded mode with degraded failure rate $\lambda_d$. It is also seen that the machines may fail simultaneously due to common cause such as frequent power-cut or shock voltage fluctuation or short-circuit, etc. according to Poisson process with rate $\lambda_c$.

For computing the steady state probabilities and other performance measures of the above defined system, we consider homogeneous set of repair rate as $\mu_1 = 2\mu, \mu_2 = 2\mu$ and $\mu_3 = 2\mu$ and take $\lambda_d = 0.6, \lambda_c = 0.05, \lambda = 0.2, \mu = 4, \alpha = 0.2, \alpha_1 = 0.1\alpha, \alpha_2 = 0.2\alpha, \alpha_3 = 0.3\alpha, \nu = 0.2$. The infinitesimal generator matrix $\bar{Q}$ is constructed as follows:

$$
\bar{Q} = \begin{bmatrix}
A_0 & B_0 & 0 & 0 \\
C_1 & A_1 & B_1 & 0 \\
0 & C_2 & A_2 & B_2 \\
0 & 0 & C_3 & A_3
\end{bmatrix},
$$

where $A_0, A_1, A_2, A_3, B_0, B_1, B_2, C_1, C_2$ and $C_3$ are sub matrices which are evaluated as follows:

**Sub matrices $A_0, A_1, A_2$ and $A_3$:**

For the sake of convenience, each of these sub matrices can be written as

$$A_i = L_i + D_i + U_i \text{ for } 0 \leq i \leq 3,$$

where $L_i, D_i$ and $U_i$ ($0 \leq i \leq 3$) denote the lower triangular matrix, the diagonal matrix and the upper triangular matrix, respectively of order $(21 - i)$ each.

Let us denote $L_j = [a_{jk}]$ for $j = 2, 3, ..., 21 - i; k = 1, 2, ..., 20 - i; U_j = [b_{jk}]$ for $j = 1, 2, ..., 21 - i; k = 2, 3, ..., 21 - i$ and $D_j = [d_{jk}]$ for $j, k = 1, 2, ..., 21 - i$. The corresponding elements of matrices $L_j, D_j$ and $U_j$ ($0 \leq i \leq 3$) are as follows:

(i) $L_0 = [a_{jk}]: a_{jk} = 0$ for $j = 2, 3, ..., 21; k = j - 1;$(ii) $L_1 = [a_{jk}]: a_{jk} = 8.00$ for $j = 2, 3, ..., 20; k = j - 1;$(iii) $L_2 = [a_{jk}]: a_{jk} = 16.00$ for $j = 2, 3, ..., 19; k = j - 1;$(iv) $L_3 = [a_{jk}]: a_{jk} = 24.00$ for $j = 2, 3, ..., 18; k = j - 1;$.
Threshold N-Policy for Degraded Machining System

(v) $D_0 = [d_{jk}]$: $d_{11} = -2.49, d_{22} = -2.47, d_{33} = -2.05, d_{44} = -2.01, d_{55} = -2.97, d_{66} = -2.93,
    d_{77} = -2.89, d_{88} = -2.83, d_{99} = -2.77, d_{1010} = -2.71, d_{1111} = -6.60, d_{1212} = -6.0,
    d_{1313} = -5.40, d_{1414} = -4.80, d_{1515} = -4.20, d_{1616} = -3.60, d_{1717} = -3.00,
    d_{1818} = -2.40, d_{1919} = -1.80, d_{2020} = -1.20, d_{2121} = -0.60.

(vi) $D_1 = [d_{jk}]$: $d_{11} = -10.47, d_{22} = -10.45, d_{33} = -10.41, d_{44} = -10.37, d_{55} = -10.33,
    d_{66} = -10.69, d_{77} = -10.23, d_{88} = -10.57, d_{99} = -10.51, d_{1010} = -14.40,
    d_{1111} = -13.80, d_{1212} = -13.20, d_{1313} = -12.60, d_{1414} = -12.00, d_{1515} = -11.40,
    d_{1616} = -18.00, d_{1717} = -10.20, d_{1818} = -9.60, d_{1919} = -9.00, d_{2020} = -8.40.

(vii) $D_2 = [d_{jk}]$: $d_{11} = -10.45, d_{22} = -18.41, d_{33} = -18.37, d_{44} = -18.33, d_{55} = -18.29,
    d_{66} = -18.23, d_{77} = -18.17, d_{88} = -18.11, d_{99} = -22.20, d_{1010} = -21.60,
    d_{1111} = -21.00, d_{1212} = -20.40, d_{1313} = -19.80, d_{1414} = -19.20, d_{1515} = -18.60,
    d_{1616} = -18.00, d_{1717} = -17.40, d_{1818} = -16.80, d_{1919} = -16.20.

(viii) $D_3 = [d_{jk}]$: $d_{11} = -10.41, d_{22} = -26.37, d_{33} = -26.33, d_{44} = -26.29, d_{55} = -26.23,
    d_{66} = -26.17, d_{77} = -26.11, d_{88} = -30.00, d_{99} = -29.40, d_{1010} = -28.80,
    d_{1111} = -28.20, d_{1212} = -27.60, d_{1313} = -27.00, d_{1414} = -26.40, d_{1515} = -25.80,
    d_{1616} = -25.20, d_{1717} = -24.60, d_{1818} = -24.00.

(ix) $U_0 = [b_{jk}]$: $b_{12} = 2.49, b_{23} = 2.47, b_{34} = 2.45, b_{45} = 2.41, b_{56} = 2.37, b_{67} = 2.33, b_{78} = 2.29,
    b_{89} = 2.23, b_{910} = 2.17, b_{1011} = 2.11, b_{1112} = 6.00, b_{1213} = 5.40, b_{1314} = 4.80,
    b_{1415} = 4.20, b_{1516} = 3.60, b_{1617} = 3.00, b_{1718} = 2.40, b_{1819} = 1.80, b_{1920} = 1.20,
    b_{2021} = 0.60.

(x) $U_0 = [b_{jk}]$: $b_{12} = 2.47, b_{23} = 2.45, b_{34} = 2.41, b_{45} = 2.37, b_{56} = 2.33, b_{67} = 2.29, b_{78} = 2.23,
    b_{89} = 2.17, b_{910} = 2.11, b_{1011} = 6.00, b_{1112} = 5.40, b_{1213} = 4.80, b_{1314} = 4.20,
    b_{1415} = 3.60, b_{1516} = 3.00, b_{1617} = 2.40, b_{1718} = 1.80, b_{1819} = 1.20, b_{1920} = 0.60.

(xi) $U_0 = [b_{jk}]$: $b_{12} = 2.45, b_{23} = 2.41, b_{34} = 2.37, b_{45} = 2.33, b_{56} = 2.29, b_{67} = 2.23, b_{78} = 2.17,
    b_{89} = 2.11, b_{910} = 6.00, b_{1011} = 5.40, b_{1112} = 4.80, b_{1213} = 4.20, b_{1314} = 3.60,
    b_{1415} = 3.00, b_{1516} = 2.40, b_{1617} = 1.80, b_{1718} = 1.20, b_{1819} = 1.60.

(xii) $U_0 = [b_{jk}]$: $b_{12} = 2.41, b_{23} = 2.37, b_{34} = 2.33, b_{45} = 2.29, b_{56} = 2.23, b_{67} = 2.17, b_{78} = 2.11,
    b_{89} = 6.00, b_{910} = 5.40, b_{1011} = 4.80, b_{1112} = 4.20, b_{1213} = 3.60, b_{1314} = 3.00,
    b_{1415} = 2.40, b_{1516} = 1.80, b_{1617} = 1.20, b_{1718} = 0.60.
Sub matrices $B_0$, $B_1$ and $B_2$ :

$B_{00}, B_{20}$ and $B_{30}$ are zero matrices of order $2 \times 1$, $2 \times 19$ and $19 \times 1$, respectively; $B_{11}, B_{21}$ and $B_{31}$ are zero matrices of order $5 \times 4$, $5 \times 15$ and $15 \times 4$, respectively; $B_{12}, B_{22}$ and $B_{32}$ are zero matrices of order $8 \times 7$, $8 \times 11$ and $11 \times 7$, respectively.

$B_{40} = [\text{diag}(0.6)]_{19 \times 19}$; $B_{41} = [\text{diag}(0.4)]_{15 \times 15}$; $B_{42} = [\text{diag}(0.2)]_{11 \times 11}$.

Sub matrices $C_1, C_2$ and $C_3$ :

$C_i = [c_{jk}]$ for $j = 1, 2, ..., 21 - i; k = 1, 2, ..., 22 - i (1 \leq i \leq 3)$ where $c_{11} = 8$ and $c_{jk} = 0$ for all $j, k \neq 1$.

Furthermore, the steady state probabilities have been computed by coding a program in MATLAB software based on matrix-recursive method as shown in Table 1.

<table>
<thead>
<tr>
<th>$i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.1154</td>
<td>0.1163</td>
<td>0.0942</td>
<td>0.0767</td>
<td>0.0622</td>
<td>0.0503</td>
<td>0.0406</td>
<td>0.0328</td>
<td>0.0264</td>
</tr>
<tr>
<td>0</td>
<td>0.1154</td>
<td>0.1163</td>
<td>0.0942</td>
<td>0.0767</td>
<td>0.0622</td>
<td>0.0503</td>
<td>0.0406</td>
<td>0.0328</td>
<td>0.0264</td>
</tr>
<tr>
<td>1</td>
<td>0.0408</td>
<td>0.0328</td>
<td>0.0068</td>
<td>0.0039</td>
<td>0.0020</td>
<td>0.0001</td>
<td>0.0006</td>
<td>0.0013</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.0304</td>
<td>0.0233</td>
<td>0.0027</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.0379</td>
<td>0.0292</td>
<td>0.0041</td>
<td>0.0041</td>
<td>0.0041</td>
<td>0.0041</td>
<td>0.0041</td>
<td>0.0041</td>
<td>0.0041</td>
</tr>
</tbody>
</table>


8. Sensitivity Analysis

MATLAB software has been used to develop the computer program for numerically analyzing the system performance under the proposed threshold $N$-policy. In this section, we first compute the performance measures for the model using the analytical results and then we compare the analytical results with the neuro-fuzzy results by building Artificial Neuro-Fuzzy Inference System (ANFIS) in MATLAB 6.5. It is worth-mentioning that soft computing approach provides flexible information processing capabilities for handling real life ambiguous situations for which classical analytical tools are difficult to use. That’s why it’s a very useful technique to solve complex problems.

For computation purpose, we consider homogeneous $(\mu_1 = 2\mu, \mu_2 = 2\mu, \mu_3 = 2\mu)$ and heterogeneous $(\mu_1 = \mu, \mu_2 = 2\mu, \mu_3 = 3\mu)$ sets of repair rates. We set default parameters as $\lambda = 0.6, \delta = 0.05, \lambda = 0.2, \mu = 4, \alpha = 0.2, \alpha_1 = 0.1\alpha, \alpha_2 = 0.2\alpha, \alpha_3 = 0.3\alpha, \nu = 0.2, \delta_1 = 2, \delta_2 = 4, \delta_3 = 3, S = \delta_1 + \delta_2 + \delta_3, M = 10, L = 20$. We have constructed Tables 2-4 and Figures 3-5 to illustrate the effect of different parameters on various performance measures.

(a) Effect of vacation rate and repair rate:

It is clear from Tables 2-3 that $E[O]$, $E[S]$ and $M.A.$ increase while $E[B]$ decreases on increasing $v$ and $\mu$ for both homogeneous and heterogeneous repair rates. It is also noted
from these figures that \( E[V] \) decreases (increases) with the increase in \( \nu (\mu) \) for both sets of repair rates. These results tally profoundly with the existing results.

Table 2. Effect of \( \nu \) on various performance measures for set 1 (homogeneous) and set 2 (heterogeneous) of repair rates.

<table>
<thead>
<tr>
<th>Repair rates</th>
<th>( \nu )</th>
<th>( E[O] )</th>
<th>( E[S] )</th>
<th>( E[B] )</th>
<th>( E[V] )</th>
<th>( M.A. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>.1</td>
<td>8.25</td>
<td>4.09</td>
<td>1.12</td>
<td>2.44</td>
<td>0.63</td>
</tr>
<tr>
<td>(Set 1)</td>
<td>.2</td>
<td>9.50</td>
<td>5.76</td>
<td>1.02</td>
<td>2.26</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>.3</td>
<td>9.84</td>
<td>6.64</td>
<td>0.94</td>
<td>2.15</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>.4</td>
<td>9.94</td>
<td>7.13</td>
<td>0.88</td>
<td>2.06</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>9.98</td>
<td>7.43</td>
<td>0.84</td>
<td>2.00</td>
<td>0.90</td>
</tr>
<tr>
<td>Heterogeneous</td>
<td>.1</td>
<td>8.25</td>
<td>3.71</td>
<td>1.33</td>
<td>2.22</td>
<td>0.62</td>
</tr>
<tr>
<td>(Set 2)</td>
<td>.2</td>
<td>9.47</td>
<td>5.42</td>
<td>1.22</td>
<td>2.08</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>.3</td>
<td>9.83</td>
<td>6.26</td>
<td>1.14</td>
<td>2.00</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>.4</td>
<td>9.95</td>
<td>6.71</td>
<td>1.08</td>
<td>1.94</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>9.99</td>
<td>6.96</td>
<td>1.04</td>
<td>1.89</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 3. Effect of \( \mu \) on various performance measures for set 1 (homogeneous) and set 2 (heterogeneous) of repair rates.

<table>
<thead>
<tr>
<th>Repair rates</th>
<th>( \mu )</th>
<th>( E[O] )</th>
<th>( E[S] )</th>
<th>( E[B] )</th>
<th>( E[V] )</th>
<th>( M.A. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>1</td>
<td>9.33</td>
<td>4.03</td>
<td>1.60</td>
<td>1.50</td>
<td>0.72</td>
</tr>
<tr>
<td>(Set 1)</td>
<td>2</td>
<td>9.48</td>
<td>5.30</td>
<td>1.26</td>
<td>2.03</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9.50</td>
<td>5.60</td>
<td>1.10</td>
<td>2.18</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.50</td>
<td>5.76</td>
<td>1.02</td>
<td>2.26</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9.50</td>
<td>5.86</td>
<td>0.98</td>
<td>2.31</td>
<td>0.79</td>
</tr>
<tr>
<td>Heterogeneous</td>
<td>1</td>
<td>8.93</td>
<td>3.04</td>
<td>1.69</td>
<td>1.15</td>
<td>0.66</td>
</tr>
<tr>
<td>(Set 2)</td>
<td>2</td>
<td>9.39</td>
<td>4.58</td>
<td>1.61</td>
<td>1.77</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9.46</td>
<td>5.20</td>
<td>1.36</td>
<td>1.97</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.47</td>
<td>5.42</td>
<td>1.22</td>
<td>2.08</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9.48</td>
<td>5.55</td>
<td>1.13</td>
<td>2.15</td>
<td>0.78</td>
</tr>
</tbody>
</table>

(b) Effect of failure rate of both operating units and spare units:

In Table 4, we see an increasing trend in \( E[B] \) whereas a decreasing trend in \( E[V] \) and machine availability \( (M.A.) \) with the increase in either \( \lambda \) or \( \alpha \). This situation can be visualized in many machining systems.

(c) Comparison of analytical results with neuro-fuzzy results:

The ANFIS and neuro-fuzzy techniques are the most powerful soft computing techniques and can also be employed for performance prediction of queueing models. The performance measures are obtained by varying parameters namely failure rate of operating and spare units \( (\lambda \text{ and } \alpha) \), repair rate of operating units \( (\mu) \) and vacation rate \( (\nu) \). These parameters are treated as the linguistic variables in the contexts of the fuzzy systems. While building the respective inference system, these parameters are taken as the input values. The gaussian function is used for describing the membership functions for these input parameters. The linguistic values of the membership functions are defined in Table 5.

The shapes of the corresponding membership functions are displayed in Figures 2 (a)-(b). A comparative study of analytical results and neuro-fuzzy results has been done in Figures 3-5.
Table 4. Effect of λ and α on some performance measures for set 1 (homogeneous) and set 2 (heterogeneous) of repair rates.

<table>
<thead>
<tr>
<th>Failure rates</th>
<th>Set 1</th>
<th></th>
<th></th>
<th>Set 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Set 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.2</td>
<td>1.02</td>
<td>2.26</td>
<td>0.78</td>
<td>1.22</td>
<td>2.08</td>
<td>0.77</td>
</tr>
<tr>
<td>.3</td>
<td>1.15</td>
<td>2.26</td>
<td>0.72</td>
<td>1.42</td>
<td>2.03</td>
<td>0.72</td>
</tr>
<tr>
<td>.4</td>
<td>1.26</td>
<td>2.24</td>
<td>0.68</td>
<td>1.57</td>
<td>1.95</td>
<td>0.67</td>
</tr>
<tr>
<td>.5</td>
<td>1.35</td>
<td>2.20</td>
<td>0.66</td>
<td>1.68</td>
<td>1.87</td>
<td>0.64</td>
</tr>
<tr>
<td>.6</td>
<td>1.44</td>
<td>2.15</td>
<td>0.64</td>
<td>1.75</td>
<td>1.80</td>
<td>0.62</td>
</tr>
<tr>
<td>Heterogeneous</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Set 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.18</td>
<td>2.28</td>
<td>0.72</td>
<td>1.48</td>
<td>2.03</td>
<td>0.72</td>
</tr>
<tr>
<td>1.2</td>
<td>1.21</td>
<td>2.28</td>
<td>0.71</td>
<td>1.53</td>
<td>2.01</td>
<td>0.71</td>
</tr>
<tr>
<td>1.4</td>
<td>1.24</td>
<td>2.27</td>
<td>0.70</td>
<td>1.58</td>
<td>1.99</td>
<td>0.70</td>
</tr>
<tr>
<td>1.6</td>
<td>1.27</td>
<td>2.26</td>
<td>0.69</td>
<td>1.62</td>
<td>1.97</td>
<td>0.70</td>
</tr>
<tr>
<td>1.8</td>
<td>1.30</td>
<td>2.25</td>
<td>0.69</td>
<td>1.65</td>
<td>1.95</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 5. Linguistic values of the membership functions for various input parameters.

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Number of membership functions</th>
<th>Linguistic values</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>4</td>
<td>Low, Average, High, Very high</td>
</tr>
<tr>
<td>α</td>
<td>4</td>
<td>Low, Average, High, Very high</td>
</tr>
<tr>
<td>ν</td>
<td>5</td>
<td>Very low, Low, Average, High, Very high</td>
</tr>
<tr>
<td>μ</td>
<td>5</td>
<td>Low, Below average, Average, Above average, High</td>
</tr>
</tbody>
</table>

Figure 2. Membership functions for input parameters (a) λ and α, (b) ν and μ, for Figures 3, 4 and 5.
In Figures 3-5, the black continuous (discrete) lines show the analytical results whereas grey continuous (discrete) lines depict the ANFIS results for homogeneous (heterogeneous) repair rates. We observe from Figures 3 (a)-(d) that $E[N]$ increases on increasing $\lambda$ and $\alpha$ but it decreases with an increase in $\nu$ and $\mu$, for both homogeneous and heterogeneous repair rates. It is also noted from Figures 4 (a)-(b) and 5 (a)-(b) that both $E[O]$ and $E[S]$ decrease on increasing $\lambda$ and $\alpha$. It is seen that the results determined by soft computing technique ANFIS are at par with the numerical results obtained analytically.

![Graphs showing analytical and ANFIS results of $E[N]$ vs $\lambda$, $\alpha$, $\nu$, and $\mu$ for set 1 (homo.) and set 2 (hetero.) of repair rates.]

Finally we conclude that

- The expected number of failed machines in the system increases with an increase in the failure rates of both operating units and spare units whereas it decreases on increasing the repair rate and vacation rate. This result matches with realistic situations.

- The expected number of operating units, spare units and machine availability increase (decrease) on increasing repair rate and vacation rate (failure rates of operating and spare units), as per our expectation.

- As expected, the expected number of busy servers and the vacationing servers in the system increase (decrease) with an increase in failure rates of operating units and spare units (repair rate and vacation rate).
Adaptive Network-based Fuzzy Interference Systems (ANFIS) provide an easy and fast solution and can be easily implemented to real time system.

9. Conclusions

In the present paper, we have considered a machining system wherein if the queue of failed machines reaches a prescribed size, the repairmen (servers) can go for multiple vacations of random length. This situation is beneficial to the system engineers as it is helpful in reducing the burden on the system and in making proper utilization of its resources which can be observed in manufacturing and production systems, distribution and transportation systems, computer and communication systems, etc. The analysis of machining systems with server’s vacation according to a threshold policy helps us to study the impact of secondary jobs during vacation and to understand and plan more realistically a real congestion situation than the case without giving due consideration to the secondary jobs. The concept that the heterogeneous servers may be employed in the system one by one depending on the queue length can be helpful to the decision makers to design a cost effective system.
Effective system. Spares are also necessary for the smooth functioning of the machining system. The application of multiple type of spares can be seen in environmental engineering systems. The provision of threshold $N$-policy is most common among many control policies for queueing systems. The incorporation of common cause and degraded failure rates leads our model to deal with embedded engineering systems including electronic/electrical, computer and communication systems, etc. The analytical results obtained in this paper are also validated by using a soft computing approach using Adaptive Network-based Fuzzy Interference System (ANFIS), which shows the computational tractability of the complex systems working in machining environment via a non-traditional way.

References


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