Optimization of an Accelerated Step-Stress Fatigue Test Plan

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Abstract: The design of a component subjected to fatigue requires an experimental qualification of the couple material/process fatigue characteristics. The current industrial requirements involve that this qualification has to be carried out on the smallest possible number of tests and within the smallest possible testing time to censorship. In the same time, the accuracy of estimations has to remain significant. To do this, accelerated step-stress fatigue tests are quite efficient. Moreover, the accuracy of this kind of test can be optimized thanks to a good distribution of the steps. This paper illustrates on a particular example the process to obtain an optimized test plan by this way. Its efficiency is compared to that obtained with other kinds of plans and the advantage of this method is shown out for the design of a component. Once the advantage of a step-stress fatigue test explained, the optimization process used to define an efficient test profile is explained. The efficiency can be evaluated by stochastic simulations or real tests defined with an optimized profile. To show out the applicability of the method and to evaluate its efficiency, the method will be carried out to design an actual mechanical component (a window mechanism) with a reliability approach.

Keywords: Accuracy, fatigue, optimization, reliability, SSALT.

1. Introduction

The evolution of the industrial competition requires designing mechanical components faster and faster. This requirement is particularly difficult to fulfill when the fatigue strength has to be qualified because, the stress levels subjected to the component are often quite low. For this reason, there is hardly any evolution of the failure probability during the first thousands of cycles. Thus, the early stage of the test is useless. Moreover, to take into account the stochastic behavior of the material, probabilized SN curves have to be drawn, which often requires accurate information about the standard deviations, and thus, a lot of specimens. All these difficulties lead to extensive costs and testing times. To face these problems, some works [8, 9] have suggested using Accelerated Life Testing (ALT). To prevent the useless testing times in the beginning of the test, the key is to increase the stress level. The first kind of ALT consists of testing the specimens at several high stress levels and to extrapolate the fatigue lives obtained to lower levels (Figure 1).

More recently, Elsayed has shown that step-stress ALTs are more efficient [4]. This second kind of ALT consists of (Figure 2) testing all the specimens at a high stress level during a given number of cycles. Then, the survival specimens are tested at another level and so on. A number of cycles to censorship may be defined to limit the testing time.
Of course, these tests require preventing any modification of the damaging mode (from the low level to the higher levels chosen). Moreover, the median fatigue life has to be expressed with respect to the stress levels by a parametric relationship called acceleration model. The usual one is the inverse power model (or Basquin's law) defined by:

\[ N(S) = \frac{a}{S^b}, \]  

where:

- \( S \) is the constant amplitude stress level,
- \( N(S) \) is the median number of cycles to failure at the constant stress level \( S \),
- \( a \) and \( b \) are the specific Basquin's parameters.

The class of statistical distribution of fatigue lives has also to be known. For example, the fatigue life at a given stress level \( S \) in the high fatigue domain is usually supposed to follow a lognormal distribution [2]. It means that the probability of non-failure at a given fatigue life \( n \) is given by:
\[
P_{NR}(n) = 1 - \Phi\left(\frac{\log(n) - \log(N(S))}{\sigma}\right),
\]

where:

- \( N(S) \) is the median fatigue life at the stress level \( S \),
- \( \Phi \) is the cumulated distribution function of the standard normal distribution,
- \( \sigma \) is the standard deviation of \( \log(n) \) at the stress level \( S \).

In the high cycle fatigue domain, \( \sigma \) can be assumed constant for any stress level \( S \) above the fatigue limit. For the first kind of ALTs, tests’ results at high levels are fitted to estimate the parameters \( a, b \) and \( \sigma \) and then the previous relationships are used to define the fatigue life distribution at the lower stress level. As far as step-stress ALTs are concerned, the fatigue life prediction often uses damaging models such as the linear Miner model [6] but experiments have shown that it was very inaccurate and that more complicated damaging models were quite hard to use in practice [7].

For step-stress ALTs, the Sedyakin’s model [10] already used by Nikulin and Bagdonavicius [1] to process the results of a step-stress ALT seems to require fewer assumptions. It consists of defining the evolution of the probability of non-failure for the step-stress test from the evolution of this probability for each single stress level, as shown on Figure 3. This method is based on two assumptions:

1. The evolution of this probability at each step-stress changing is continuous.
2. From this probability’s value at the beginning of a given step, its evolution only depends on the stress level at this step.

At the end of any ALT, Least Square [5] or Maximum Likelihood [1] estimates of the parameters can be obtained. The estimates’ accuracy can also be estimated by the use of bootstrap methods or the covariance Fisher information matrix.

A general overview on these testing methods and on the model and statistical approach used is given in [1]. With this method, it has recently been suggested optimizing the distribution of the steps’ levels and durations [5, 6]. One has to evaluate by simulation a lot of test plans and to select the best one, which can be done by random generation of the fatigue
lives. The problem is that the number of simulations required may be very high. Thus, it may be useful to concentrate on the most potentially efficient test plans. We suggest concentrating on plans for which the number of failures observed for the different specimens is equally distributed among the different steps. To do this, the following process can be carried out:

A first estimation of parameters is obtained from short tests (tensile test, fatigue test beneath the low cycle fatigue domain). The evolution of the non-failure probability is defined at one single stress level from these first estimations. The times corresponding to several fractiles of the number of failures are calculated. From the first estimations of parameters, these times are changed into ending step times leading to the same fractiles. The step-stress plan obtained is tested by several simulations to see the influence of the stochastic behavior and to verify the robustness of this test plan. To do this, the values of parameters estimated after each simulated test at the nominal level chosen are compared to input values. When the accuracy is ensured, the real step-stress fatigue tests are carried out on the available specimens. Then, the fatigue lives obtained are processed by Maximum Likelihood Estimation (MLE) and the estimations’ accuracy is estimated too (by the Fisher information matrix).

This method can be applied to quickly qualify materials from tests on smooth specimens but it can be done on actual mechanical components too. If short tests results, bibliography, Finite Elements simulations, or prior tests’ results on similar components are available, it enables one to obtain first estimations for the component of interest, and thus to define an optimized test plan. Then, this plan can be carried out on smooth specimens made of stretched C38 steel and on an end stop window’s mechanism. The detail of the experiments and the results obtained are detailed in the following. Its efficiency is verified by comparison with a classical fatigue test plan.

2. Experiments on Smooth Specimens

2.1. Short Tests

The purpose of the short tests is:

- to identify in the bibliography the relevant information about the material used,
- to bound the high cycle fatigue domain,
- to have a first estimation of the fatigue limit $S_D$, the Basquin’s parameters and the standard deviation $\sigma$ of $\log(n)$,

where $n$ denotes the fatigue life at a given stress level in the high cycle fatigue domain.

First of all, one or more tensile tests can indicate the ultimate strength and the yield strength. Thus, the upper limit of the high cycle fatigue can be located. A hardness test enables one to verify that the limit obtained is consistent with the material hardness. The microscopic observations indicate the grain size and shape. Thus, the heat and mechanical treatments subjected by the specimens can be known. At last, some fatigue tests at a stress level $S_1$ slightly beside the yield stress $R_y$ give an estimation of $\sigma$ and a mean value $\mu_1$ for $\log(n)$ at this stress level.

The relationship (1) represents a linear evolution of $\log(n)$ with respect to $S$. Thus, this evolution can be plotted by a straight line joining the two points ($\mu_1, S_1$) and ($\log(10^3, S_D)$) and then, the estimation of the Basquin’s parameters $a$ and $b$ is obvious. At the end of these short tests, approximations of all the parameters required to simulate any step-stress test in the high cycle domain are available and, the AFT model used by Bagdonavicius and Nikulin [1] may change any number of cycles $n$ for a step-stress test into...
an equivalent number of cycles \( n_{\text{equ}} \) defined for a test at a single stress level (and the inverse transformation is possible too).

Let us remind that, for two different fatigue tests, two testing numbers of cycles are told equivalent if the probabilities of failure at these two numbers of cycles are equal. Thus, the step-stress test can be defined so that the same fractile of failures is defined at all the steps and it can be verified by simulations in which random values corresponding to the step-stress test found are generated (with the estimates of \( a, b \) and \( \sigma \) obtained after the short tests). For the C38 steel used, tests on smooth specimens give the following results (Table 1).

<table>
<thead>
<tr>
<th>UTS</th>
<th>YS</th>
<th>( S_0 )</th>
<th>( S_1 )</th>
<th>log(( \mu_i ))</th>
<th>( a )</th>
<th>( b )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>810 MPa</td>
<td>605 MPa</td>
<td>305 MPa</td>
<td>545 MPa</td>
<td>9.7 ( 10^3 )</td>
<td>0.08</td>
<td>1110 MPa</td>
<td>0.08</td>
</tr>
</tbody>
</table>

2.2. Definition and Simulations of the Step-Stress Tests

From the estimations of \( \mu_i \) and \( \sigma \), the probability of non-failure at the stress level \( S_i \) can be plotted and 5 fractiles (on the number of cycles \( n \) \( n_{\beta}, i = 1 \) to 5 can be defined. These fractiles are changed into 5 final numbers of cycles \( n_{\beta}, i = 1 \) to 5 for each step \( i \) as shown on Figure 3. With the estimations in Table 1, it leads to the values in Table 2.

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_i )</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>( n_{\beta} )</td>
<td>32 050</td>
<td>33 820</td>
<td>36 240</td>
<td>44 140</td>
<td>50 500</td>
</tr>
</tbody>
</table>

Figure 4 shows how the failures on 20 simulated smooth specimens are distributed. It leads to hardly the same distribution on the different steps for 10 simulations, which shows that, despite of the scatter in the behaviour of the material, this profile often leads to a good distribution of the stress levels at the different failures. Thus, it seems that with a sample size equal to 20 and with a censorship at \( n_c = 6.10^4 \) cycles, this plan gives the best accuracy. However, it has to be verified by comparison with other plans.

To do this, the numbers of cycles \( n_{\beta}, i = 1 \) to 4 are shifted from \(-10\%, -5\%, 5\%, 10\%\) (the fifth one is unchanged because it is the number of cycles to censorship) and the same is done on the stress levels at each step. Each time, a step-stress test on 20 specimens is simulated 10 times, which leads to a mean estimation of the number of failures at \( 10^6 \) cycles and to a mean accuracy. Then, it can be confirmed that the best accuracy is found with the plan defined in Table 2, which demonstrates that it is an optimal test plan (if the estimations of \( a, b \) and \( \sigma \) are not too far from their true values and if the shifts chosen are representative enough).

2.3. Experimental Results on Smooth Specimens

The distribution of failures obtained is shown on Figure 5. Because of the slight difference between the estimated values of \( a, b \) and \( \sigma \) and their true values, the fatigue lives observed are not exactly distributed among all the steps but it is hardly what was expected anyway.
To evaluate the efficiency of that plan, maximum likelihood (ML) estimates can also be obtained from the usual plan on 30 specimens subjected to 5 constant amplitude stress levels [6]. The ML estimates obtained from these real results can be compared to those obtained from simulated results. Moreover, from the Fisher information matrix, the coefficients of variation (CoV) estimates, indicating the relative errors on the point estimates, can be compared too (Table 3).

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$ (MPa)</th>
<th>$\sigma$</th>
<th>$S$ ($10^6$ cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates from short tests</td>
<td>0.08</td>
<td>1110</td>
<td>0.08</td>
<td>368</td>
</tr>
<tr>
<td>Estimates from the usual plan (reference)</td>
<td>0.143</td>
<td>2053</td>
<td>0.83</td>
<td>285</td>
</tr>
<tr>
<td>Estimates from the step-stress test</td>
<td>0.14</td>
<td>2059</td>
<td>0.95</td>
<td>298</td>
</tr>
<tr>
<td>CoV estimates (%)</td>
<td>4.5</td>
<td>3.7</td>
<td>17.5</td>
<td>7.2</td>
</tr>
</tbody>
</table>

It leads to the evolution of the sure curves below (defined at the mean $\log n$ minus 3 standard deviations).
Figure 4. Distribution of the failures for the optimized test plan (15 failures, 5 censored).

Figure 5. Results of a real optimized step-stress test.

To evaluate the efficiency of that plan, maximum likelihood (ML) estimates can also be obtained from the usual plan on 30 specimens subjected to 5 constant amplitude stress levels [6]. The ML estimates obtained from these real results can be compared to those obtained from simulated results. Moreover, from the Fisher information matrix, the coefficients of variation (CoV) estimates, indicating the relative errors on the point estimates, can be compared too (Table 3).

Table 3. Estimations and accuracy level.

<table>
<thead>
<tr>
<th></th>
<th>Estimates from short tests</th>
<th>Estimates from the usual plan (reference)</th>
<th>Estimates from the step-stress test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$ (MPa)</td>
<td>0.08</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>$V_6(10^{6}\text{s cycles})$</td>
<td>1110</td>
<td>2053</td>
<td>2059</td>
</tr>
<tr>
<td>$\text{CoV}$ estimates (%)</td>
<td>4.5</td>
<td>3.7</td>
<td>7.2</td>
</tr>
</tbody>
</table>

It leads to the evolution of the sure curves below (defined at the mean minus 3 standard deviations).

Figure 6. Lower bounds of $\log n$ with a 99.3% confidence level.

Not only the fatigue strength at $10^6$ cycles estimated from the optimized plan is close to that estimated from the usual test plan, but also, the estimate of the CoV enables one to correctly bound the true value (as the usual tests are carried out on a lot of specimens, the corresponding estimation can be considered as a reference). It can be seen that the important errors on the estimates obtained from short tests do not prevent a good efficiency of the optimized plan, which demonstrates the robustness of the method.

3. Application to Mechanical Components

The component tested is a guideway used for pivoting windows. The weak point particularly subjected to fatigue is the guideway stop end (Figure 7). In this study, the optimized test plan can be defined from an existing fatigue curve obtained with a usual fatigue test plan at constant amplitude force levels. It had been done by pushing on the guideway stop end with an axial fatigue test machine. These results show that the evolution of the pressing force with respect to the corresponding number of cycles to failure has the same shape as the SN curve for steel smooth specimens. The central region can also be modelled by a Basquin’s relationship, and the standard deviation of $\log(n)$ at a given force level can be supposed constant in this region. Thus, from the parameters obtained, an optimized test plan with successive steps at 5 different force levels could be defined.

Figure 7. Guideway with its stop end.
This test can be carried out on 5 components only, but in this case, a Bayesian approach is required. It means that prior information has been used on the parameters to eliminate the unavoidable effect of scatter for such a small sample. In our case, the a priori is a uniform distribution of the three parameters from 80% to 120% of the usual estimate obtained on a similar model made of the same kind of steel. The application of the optimized test plan method can be sum up as follows.

1. From old tests on a similar model of guideway, the prior estimates of \( a, b \) and can be defined by flat distributions (for simplicity) on their usual 95% confidence intervals, namely \( 0.08 \pm 10\% \), \( 9250 \text{N} \pm 15\% \) and \( 0.4 \pm 25\% \) (the Basquin's relationship is correct from 2000 to 5000N).

2. To distribute the failures on all the steps and to ensure the relationship validity, the step-stress test has been defined with the 3 decreasing force amplitude levels: \( F_1 = 4000 \text{N}, F_2 = 3200 \text{N} \) and \( F_3 = 2400 \text{N} \).

3. The number of cycles at the end of each of these steps is defined for fractiles of failures equal to 0.3, 0.6 and 0.9. A test at the constant force amplitude level \( F_1 \) leads to the numbers of cycles: 11,020, 17,550 and 24,850.

The method leads to the step-stress test defined with the numbers of cycles at the end of each step: 11,000, 118,000 and 4.5 \( \times 10^6 \). In fact, the last one is replaced by the number of cycles to censorship \( n_k = 4 \times 10^6 \). Simulations of ALT tests with slightly different values of \( n_k \) do not lead to more accurate ML estimates.

Of course, it would be better validating the interest of the step-stress fatigue tests chosen for a metal fatigue qualification or for the validation of a whole mechanical component whereas it was correct. Specimens demonstrate that the method is not only theoretically justified but also practically satisfactory. The experiments carried out on specimens subjected to fatigue, the optimization method presented enables one to save time and money without reducing the accuracy of fatigue lives estimations. The experiments carried out on 100 components subjected to fatiguing at 10\(^7\) cycles are \( F_1 = 2430 \text{N} \) with a CoV equal to 43\% and an standard deviation's estimate on \( \log(n) \): \( \hat{\sigma} = 0.27 \) with a CoV equal to 82\%.

The strong inaccuracy on these estimates stems from the very little size of the tested sample. Better estimates can be obtained by the use of the Bayesian prior estimates defined above. It leads to the posterior estimates: \( \hat{a} = 8.43 \times 10^2, \hat{b} = 9735 \text{N}, \hat{F}_2 = 2509 \text{N} \) and \( \hat{\sigma} = 0.384 \).

As a Bayesian method is used, the accuracy cannot be estimated by the Fisher information matrix anymore but it can be estimated from several test simulations. Thus, 100 lists of 5 fatigue lives can be defined by random generations with the parameters estimated from the actual tests. Each time, new estimates of these parameters are obtained with the same Bayesian method and with the same prior estimates. Because of the random effects of this process, the posterior estimates obtained are always different from the input values. As a comparison, the classical plan of fatigue tests can be carried out with the constant force amplitudes \( F_1, F_2 \) and \( F_3 \) and with the same sample size and number of cycles to censorship. ML and Bayesian estimates obtained and the corresponding CoV for 100 test simulations are sum up in Table 4.

Table 4. Comparison of estimates with the classical plan and with the optimized plan (with the CoV).

<table>
<thead>
<tr>
<th></th>
<th>( a )</th>
<th>( b ) (MPa)</th>
<th>( F_2(10^7) ) in N</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref: Usual plan on 100 guideways</td>
<td>0.086 ± 0.8%</td>
<td>9732 ± 2%</td>
<td>2503 ± 0.3%</td>
<td>0.39 ± 8%</td>
</tr>
<tr>
<td>MLE with the classical plan - 5 guides</td>
<td>0.078 ± 11%</td>
<td>9248 ± 58%</td>
<td>2107 ± 55%</td>
<td>0.19 ± 230%</td>
</tr>
<tr>
<td>MLE with the optimized plan - 5 guides</td>
<td>0.079 ± 6%</td>
<td>9540 ± 39%</td>
<td>2430 ± 43%</td>
<td>0.27 ± 82%</td>
</tr>
<tr>
<td>Bayesian estim. (classical plan) - 5 gu.</td>
<td>0.082 ± 5%</td>
<td>9726 ± 5%</td>
<td>2478 ± 3%</td>
<td>0.412 ± 21%</td>
</tr>
<tr>
<td>Bayesian estim. (optimized plan) - 5 gu.</td>
<td>0.085 ± 1%</td>
<td>9735 ± 2%</td>
<td>2509 ± 0.4%</td>
<td>0.384 ± 11%</td>
</tr>
</tbody>
</table>
If the reference is the estimation of parameters obtained with 100 components subjected to constant force amplitudes and without any Bayesian approach, it is shown that, for the same prior estimates and the same sample sizes, the posterior estimates obtained from the optimized plan are more accurate than those obtained with constant force amplitudes. The efficiency of an optimized SSALT for Bayesian estimates is shown on Figure 8.

These acceptable results require relevant prior estimates. For our study, it is possible if a similar model of guideway is tested before but such results are not always available. When this is the case, the prior estimates may also be obtained from finite elements simulations with the fatigue parameters of the material used, or with short test results or also with values available in bibliography. Otherwise, the uncertainty on the priors may be taken into account by using their lower and upper bounds for the calculations.

4. Conclusion

For a metal fatigue qualification or for the validation of a whole mechanical component subjected to fatigue, the optimization method presented enables one to save time and money without reducing the accuracy of fatigue lives estimations. The experiments carried out on steel specimens demonstrate that the method is not only theoretically justified but also applicable for industrials.

Of course, it would be better validating the interest of the step-stress fatigue tests chosen for other kinds of metals such as aluminium or copper alloys. However, the only caution to take to do this is to verify the accuracy of the Sedyakin and Basquin's models. This can be done quite conveniently by preventing any plastic strain during the test (with measures of temperatures for example) and anyway, it is possible to control at the end of the tests if the final results are consistent with these models.

As far as tests on whole mechanical components are concerned, this method is particularly useful because, for the moment, it is very hard to obtain accurate estimations of standard deviations with the usual sample size available. It leads to overestimate the standard deviation and thus to over dimension the component. Sometimes, it requires redesigning the component whereas it was correct.
As a conclusion, it is worth designing an optimized fatigue test plan fitted to the tested material or component. Despite of the innovating aspect of the method, there is very little chance to misevaluate the final fatigue strength and, anyway, the inaccuracy can be known to decide if testing plan should be completed with other tests on other specimens.

References

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