Optimal Decisions on the Accelerated Degradation Test Plan Under the Wiener Process

Tzong-Ru Tsai¹, Y. L. Lio²* and Nan Jiang²
¹Department of Statistics, Tamkang University, New Taipei City, Taiwan
²Department of Mathematical Science, University of South Dakota, Vermillion, SD, USA

(Received March 2013, accepted July 2013)

Abstract: The cumulative damage of highly reliable products that subject to multiple loading stresses is investigated under Wiener process. Optimal strategies on the constant-stress accelerated degradation test plan are established to reach a compromised decision between the experiment budget and the estimation precision on the reliability inference. An algorithm is provided to search an optimal strategy for the accelerated degradation test. An example of light emitting diodes is used for illustrating the application of the proposed method.

Keywords: Accelerated life test, cumulative exposure model, Fisher information, maximum likelihood estimation, generalized Eyring model.

1. Introduction

Accelerated degradation tests, ADTs, are often used to assess the lifetime quality of highly reliable products which are not likely to fail by using traditional life tests such as censoring tests, truncated life tests or accelerated life tests. Topics of using the constant-stress or step-stress ADT method, labeled by CSADT and SSADT, respectively, for the reliability inference have been studied by many authors over the past few decades, for example, Liao and Elsayed [3], Lim and Yum [4], Lu and Meeker [6], Padgett and Tomlinson [10], Park and Padgett [12-15], Tsai et al. [17], Tsai et al. [18], Tseng et al. [19], Tseng and Peng [20], Tseng et al. [21] and Tseng and Wen [22].

The SSADT model has obtained tremendous attention due to the benefit of saving resources. Inferences on lifetime distribution parameters via step-stress degradation data are derived based on the assumption of cumulative exposure model, CEM. The CEM specifically assumes that the residual life of a test unit only depends upon the cumulative exposure of the unit under the currently set-up stress condition without the memory of how this exposure to be accumulated; See Komori [1] and Miller and Nelson [7]. Tsai et al. [18] mentioned that the quality degradation of highly reliable products could not be well controlled under higher stress loading conditions in an ADT. Hence, the assumption of CEM cannot be validated. It becomes unrealistic to ignore the exposure effect that has cumulated from the former set-up stress conditions under the step-stress loading design. Therefore, the CEM assumption would be doubtful and might make the reliability inference of highly reliable products inaccurate.

To conduct an ADT, a well planned strategy is necessary to reach a compromised decision between the experiment budget and the estimation precision on the reliability

* Corresponding author. E-mail: Yuhlong.Lio@usd.edu
inference. Some recent studies about the optimal ADT design methods can be found in Liao and Tseng [2], Lim and Yum [4], Liu and Tang [5], Peng [16], Tsai et al. [18], Tseng et al. [19], Tseng et al. [21] and Ye et al. [23]. Onar and Padgett [8] suggested a general approach, which is based on locally penalized D-optimality, called LPD-optimality, for ADT designs. The LPD-optimality approach can simultaneously minimize the variances of the model parameter estimators. Using Bayesian Markov chain Monte Carlo method, Pan and Balakrishnan [11] provided an efficient estimation method to obtain maximum-likelihood estimates, MLEs, of the multiple-step SSADT models based on Wiener and gamma processes, respectively. The ADT designs studied by Onar and Padgett [9] and Pan and Balakrishnan [11] were developed under one accelerating variable or step-stress loading model. However, highly reliable products are often subject to multiple loading stresses and a pitfall in SSADT model is the memoryless assumption. To incorporate several accelerating variables into ADT methods, Park and Padgett [14-15] provided hyper-cuboidal volume approaches for different CSADT models with several accelerating variables. But their methods do not cover the interaction effect of stress variables. In reality, temperature and current are two common accelerating variables for ADTs to accelerate the failure of the test units. Tsai et al. [18] proposed an inference approach with the CSADT to incorporate both accelerating variables of temperature and current, as well as the interaction effect for the lumen degradation of light emitting diodes, LEDs, under Wiener process. Based on the inference procedure suggested by Tsai et al. [18], planning ADT strategies under the CSADT model are proposed in this paper such that the asymptotic variance of the 100\(p^{th}\) lifetime percentile estimator is minimized subject to a budget constraint. The rest of this paper is organized as follows: two examples are provided in Section 2 to motivate the necessity for developing an optimal ADT strategy. In Section 3, optimal designs on CSADT are established by the maximum-likelihood estimation method and Fisher information. Moreover, an algorithm is given to search the proposed optimal ADT strategy. An example regarding LED products is used in Section 4 for illustrating the application of the proposed method. Finally, some concluding remarks are given in Section 5.

2. The Motivating Examples

A lumen degradation data set of transistor outline can (TO-can) packaged LEDs has been investigated by Tsai et al. [18]. This data set was collected from 2010 to 2011 in a laboratory of Taiwan via an ADT with accelerating variables, the absolute ambient temperature in Celsius degree (\(^{o}\)C) and the driven current in milliampere (mA). Measurements of luminous flux of LED source were collected using KEITHLEY 2430 pulse source current meter with integrating sphere OL500 and a spectroradiometer CAS140B during the ADT.

The normal use condition was set at 25 \(^{o}\)C and 350mA for LED units. Six experimental runs with the following stress loading combinations of two variables, (25, 350), (45, 650), (60, 650), (75, 450), (75, 550) and (75, 650) in (\(^{o}\)C, mA), have been used for the ADT. Using this data set, Tsai et al. [18] provided an interval estimation method under Wiener process. However, the planning ADT strategy in their study is subjective. In practice, an ADT is often implemented to meet the time schedule for the products introduced into the marketplace with an optimal level of the estimation precision from the reliability inference that subject to a given experiment budget. Engineers would like to reach a compromised ADT strategy subject to the budget constraint such that the estimation precision from the reliability inference is optimized. Because of these reasons, planning ADT strategies need to be suitably developed for LED products.
The developed ADT planning strategy can also be used to evaluate the reliability of other highly reliable products that subject to two loading stresses, for example, to study the reliability of battery products. It is known that the battery lifetime depends upon the moisture, temperature and material. The moisture and temperature could be considered to be two accelerating variables for the ADT experiment to evaluate the reliability of battery products. Planning ADT strategies need to be suitably developed for battery products.

3. Methodology

3.1. Maximum-Likelihood Estimation

In this study, two stress loading variables, labeled by \( L_1^* \) and \( L_2^* \) are considered for CSADT experiments to speed up the failure process of test units. The pairs of the lowest and highest stress levels of \( L_1^* \) and \( L_2^* \) are denoted by \( (L_{11}, L_{20}) \) and \( (L_{1M}, L_{2M}) \), respectively. The degradation measurements observed at times \( t_{ij1}, t_{ij2}, \ldots, t_{ijm} \) for test unit \( j \) subject to stress loading combination \( i \) are labeled by \( x_{ij1}, x_{ij2}, \ldots, x_{ijm} \). Denote the number of test units subject to stress loading combination \( i \) by \( n_i \), and let \( n_{xt} = \sum_{i=1}^{k} n_i \). The number of stress loading combination used in the ADT is labeled by \( k \). Through the standardization processes (utilized by Lim and Yum [4]) for \( L_{1i} \) and \( L_{2i} \), respectively, defined by

\[
L_{1i} = \frac{1}{L_{1i0} - 1/L_{1iM}} \quad \text{and} \quad L_{2i} = \frac{\log(L_{2i1}) - \log(L_{2i0})}{\log(L_{2iM}) - \log(L_{2i0})}, \quad i = 1, 2, \ldots, k, \tag{1}
\]

we obtain the standardized stress levels \( L_i = (L_{1i}, L_{2i}) \) with \( L_{10} = L_{20} = 0 \), \( L_{1M} = L_{2M} = 1 \), \( 0 < L_{1i} \leq 1 \) and \( 0 < L_{2i} \leq 1 \), \( i = 1, 2, \ldots, k \). The damage measurements \( \Delta x_{ijh} = x_{ijh} - x_{ij(h-1)} \), \( h = 1, 2, \ldots, m_{ij} \), \( j = 1, 2, \ldots, q_i \), \( i = 1, 2, \ldots, k \) for each survival unit are assumed to follow a Wiener process with drift parameter \( \nu_L \) and diffusion parameter. Without loss of generality, let initial values \( t_{ij0} = 0 \) and \( x_{ij0} = 0 \) for all \( i, j \). The drift parameter can be expressed in terms of \( L_i = (L_{1i}, L_{2i}) \), \( i = 1, 2, \ldots, k \), via a generalized Eyring model, GEM, which is defined as follows:

\[
\nu_L = \exp(\gamma_0 + \gamma_1 L_{1i} + \gamma_2 L_{2i} + \gamma_3 L_{1i} L_{2i}) \quad i = 1, 2, \ldots, k. \tag{2}
\]

The lifetime of a test unit is defined as the first passage time for the cumulative damage over a given threshold \( C \). Assume that \( p_i \) failed units have been observed with lifetimes, \( s_{ij} \), \( i = 1, 2, \ldots, p_i \), respectively, in the run \( i \) and \( q_i \) units still survive at the termination time of run \( i \). Hence, \( n_i = p_i + q_i \), \( i = 1, 2, \ldots, k \). Let \( \lambda = C^2 / \beta^2 \), \( \Theta = (\gamma_0, \gamma_1, \gamma_2, \gamma_3, \beta) \) and \( \gamma_{ijh} = (C - x_{ijh})(C - x_{ij(h-1)}) / \Delta x_{ijh} \) for all \( i, j, h \). Using the properties of Wiener process and an inference process similar to that given by Tsai et al. [18], the log-likelihood function based on both failure times and degradation measurements can be represented as follows:

\[
\ell(\Theta) \propto -n_{xt} \log \beta - \sum_{i=1}^{k} \sum_{l=1}^{p_i} \left( \frac{C - \nu_L s_{il}}{2\beta^2 s_{il}} \right)^2 - \sum_{i=1}^{k} \sum_{j=1}^{q_i} \sum_{h=1}^{m_{ij}} \left( \frac{(\Delta x_{ijh} - \nu_L \Delta t_{ijh})^2}{2\beta^2 \Delta t_{ijh}} - \log 1 - e^{-\frac{2y_{ijh}}{\beta^2}} \right). \tag{3}
\]

It is challenged to find the MLEs of these five parameters, \( \gamma_0, \gamma_1, \gamma_2, \gamma_3 \) and \( \beta \), by maximizing the log-likelihood function of Equation (3). R source codes have been developed to determine the MLEs from Equation (3) via solving nonlinear likelihood
equations, obtained by using the same method as Tsai et al. [18]. The Fisher information matrix can be derived as

\[
I(\Theta) = \begin{bmatrix}
a_1 & a_2 & a_3 & a_4 & 0 \\
a_2 & a_5 & a_6 & a_7 & 0 \\
a_3 & a_6 & a_8 & a_9 & 0 \\
a_4 & a_7 & a_9 & a_{10} & 0 \\
0 & 0 & 0 & 0 & a_{11}
\end{bmatrix},
\]

where the entries \(a_i\)'s are presented in Appendix. Analogy to the inference procedure proposed by Onar and Padgett [8], the MLE of the 100\(p\)th lifetime percentile at normal use condition can be approximately obtained by

\[
\hat{s}_{pl0} = \left[ z_p\beta + \sqrt{z_p^2\beta^2 + 4Ce^{\gamma_0}} \right] / \left( 4e^{2\gamma_0} \right).
\]

where \(z_p\) is the 100\(p\)th percentile of standard normal distribution. Using the Fisher information matrix of Equation (4), the asymptotic variance of \(\hat{s}_{pl0}\) is obtained and denoted by

\[
\sigma^2(\hat{s}_{pl0}) = \kappa^T \Gamma^{-1}(\Theta) \kappa,
\]

where

\[
\kappa^T = \left[ \frac{\partial S_{pl0}}{\partial \gamma_0}, 0, 0, 0, \frac{\partial S_{pl0}}{\partial \beta} \right],
\]

\[
\frac{\partial S_{pl0}}{\partial \gamma_0} = \frac{\left[ z_p\beta + \sqrt{z_p^2\beta^2 + 4Ce^{\gamma_0}} \right] \left[ 2Ce^{\gamma_0} (z_p^2\beta^2 + 4Ce^{\gamma_0})^{1/2} \right] - \left( z_p\beta + \sqrt{z_p^2\beta^2 + 4Ce^{\gamma_0}} \right)}{2e^{2\gamma_0}},
\]

and

\[
\frac{\partial S_{pl0}}{\partial \beta} = \frac{1}{2e^{2\gamma_0}} \left[ z_p\beta + \sqrt{z_p^2\beta^2 + 4Ce^{\gamma_0}} \right] \left[ z_p \left( 1 + z_p\beta (z_p^2\beta^2 + 4Ce^{\gamma_0})^{-1} \right) \right].
\]

### 3.2. Optimal Strategy

To implement an ADT, engineers are interested in developing an optimal strategy for the sample size and termination time of each run. Denote the length of the measuring time interval for all units in the run \(i\) by \(\delta_i\), which can be determined according to the operation schedule. In practice, a constant length of the measuring time interval with \(\delta_i = \delta\) is preferred due to administrative convenience. Let the total cost for implementing an ADT be labeled by TC. Based on the ADT experience from 2010 to 2011 for collecting the LED data set used by Tsai et al [18], the TC involves three components, fixed cost, total operating cost and variable cost, that are described as follows: (i) Fixed cost is \(c_0 \times n_{\text{test}}\), where \(c_0\) is the fixed cost per unit. (ii) Total operating cost can be presented by \(c_{op} \sum_{i=1}^{k} t_i\), where \(c_{op}\) is the operating cost per unit time. (iii) The variable cost is based on the fact that using different stress loading on the test units in the laboratory incurs a different cost. It is assumed that a rise of one degree in temperature beyond the normal use loading level costs \(c_{T_i}\) dollars per unit time and a rise of one milliampere in current costs \(c_{I_i}\) dollars.
Optimal Decisions on the Accelerated Degradation Test Plan Under the Wiener Process

465

per unit time for each test unit. Thus, variable cost in the ADT experiment can be presented as $c_{v\ell} \sum_{i=1}^{k} t_{i}(L_{i} - L_{0}) + c_{p\ell} \sum_{i=1}^{k} (n_{i} \times t_{i} \times L_{0})$. In reference of (i), (ii) and (iii) for TC components, the TC can be evaluated by

$$TC = c_{0} \times n_{\text{ext}} + c_{o\ell} \sum_{i=1}^{k} t_{i} + c_{p\ell} \sum_{i=1}^{k} t_{i}(L_{i} - L_{0}) + c_{v\ell} \sum_{i=1}^{k} (n_{i} \times t_{i} \times L_{0}). \quad (6)$$

In some cases, the variable cost may contain different components from the proposed cost model, for example, the variable cost could include operating cost, labor cost and cost incurred under different stress level combinations. The total cost model needs to be re-formulated if any composition of fixed cost, operating cost or variable cost is different from the proposed one. For some experiments, the unit cost of testing electronic device could vary with different mA levels due to the difficulty to control the lower current in the mA level. It requires more sophisticated and precision instruments to generate the current and maintain the stability of the current at a lower mA level. In such cases, the unit current cost could be changed with respect to the current level. A multiple-level current cost function is suggested for such cases.

Very often, the scheduled time to finish the reliability analysis of highly reliable products via ADT is tight because product providers would like to promote products into the market as early as possible. Hence, an upper bound of experimental time, labeled by $t_{U}$, would be preassigned. Let $n = (n_{1}, n_{2}, \ldots, n_{k})$, and $\hat{\sigma}^{2}(n, t) = \hat{\sigma}^{2}(\delta, n_{\text{ext}})$. The optimal settings of $(n^*, t^*)$ can be determined such that $\hat{\sigma}(n, t)$ is minimized, subject to a given total budget, say $\phi_{0}$. It is reasonable to assume that all units in each experimental run need to be measured at least once. Then the optimal test plan $(n^*, t^*)$ can be obtained through solving System (7). Without loss of generality, let $t_{U}$ be a positive integral multiple of the length of the measuring time interval, $\delta$. The optimum scheduled experiment times for all $k$ runs can be set up such that $\delta \leq t_{1} \leq t_{2} \leq \ldots \leq t_{k} \leq t_{U}$. If an engineer would like to have the ADT containing 6 runs for the LED example in Section 2 to be implemented and ended by 36 weeks with $\delta = 6$ weeks or 1008 hours to collect the degradation information. Then $t_{U}$ is 36 weeks or 6048 hours. The optimal experimental time schedule for the ADT can be determined from the set $\{1008 \leq t_{1}, t_{2}, \ldots, t_{k} \leq 6048\}$ through solving System (7).

Minimize $\hat{\sigma}(n, t)$

Subject to

$$TC \leq \phi_{0},$$

$$\delta \leq t_{1} \leq t_{2} \leq \ldots \leq t_{k} \leq t_{U},$$

$$n_{i} \geq 1, i = 1, 2, \ldots, k,$$

$$n_{1} + \ldots + n_{k} = n_{\text{ext}}.$$

Let $B_{i}$ be the set of all possible combinations of $t$ such that the conditions of $\{\delta \leq t_{1}, t_{2}, \ldots, t_{k} \leq t_{U}\}$ and $TC \leq \phi_{0}$ are satisfied. The interval $[\delta, t_{U}]$ is divided into subintervals with equal length $\delta$. Then the partition points are $\delta, 2\delta, \ldots, t_{U}$. The collection of all possible subsets of $\{\delta, 2\delta, \ldots, t_{U}\}$ is labeled as $B_{i}$, defined by $B_{i} = \{t | \delta \leq t_{1} \leq t_{2} \leq \ldots \leq t_{k} \leq t_{U} \text{ and } TC \leq \phi_{0}\}$. Global searching from all possible solutions in order to obtain an optimal strategy on $(n^*, t^*)$ is time consuming and very difficult. In this paper, we provide an simple algorithm to find the optimal ADT plan on $(n^*, t^*)$ if the equal number of highly reliable units are allocated for all experimental runs, that is, $n_{i}^* = n^*, i = 1, 2, \ldots, k$. Such design is easily operated by engineers to implement an ADT. Based on this setting,
the optimal ADT plan \((n^*, t^*)\) can be reduced to \((n^*, t^*)\). For each \(t^{(i)} = (t^{(i)}_1, \ldots, t^{(i)}_k)\) in \(B_i\), an upper bound of sample size, denoted by \(n_{U,i}^{t^{(i)}}\), can be obtained by using the inequality, \(TC < \phi_0\), in System (7). We then have

\[
n_{U,i}^{t^{(i)}} = \left[ \phi_0 - c_{op} \sum_{j=1}^{k} t^{(i)}_j - c_{L_2} \sum_{j=1}^{k} t^{(i)}_j (U_{L_j} - L_{L_0}) \right] \frac{k \times c_0 + c_{L_2} \sum_{j=1}^{k} (t^{(i)}_j \times L_{L_2}'}{k}
\]

where \(\left\lceil x \right\rceil\) denotes the largest integer less than or equal to \(x\). Let

\[
\Psi(n^*_{t^{(i)}} | t^{(i)}) = \min_{n=1,2,\ldots,n_{U,i}^{t^{(i)}}} \tilde{\sigma}(n, t^{(i)}), \quad i = 1, 2, \ldots, n_{B_i},
\]

and

\[
\Psi(n^*, t^*) = \min_{i=1,2,\ldots,n_{B_i}} \Psi(n^*_{t^{(i)}} | t^{(i)})
\]

where \(n^*_{t^{(i)}}\) is the optimal sample size at times \(t^{(i)}\). Then the optimal ADT plan, \((n^*, t^*)\), can be obtained through the following algorithm:

**Algorithm**

1. Determine the set \(B_i\).

2. Find \(n_{U,i}^{t^{(i)}}\) and \(\Psi(n^*_{t^{(i)}} | t^{(i)})\) via Equation (8) for each \(t^{(i)}\) in \(B_i\) and Equation (9), respectively.

3. The optimal ADT plan \((n^*, t^*)\) is the solution such that Equation (10) is satisfied.

**4. An Illustrative Example**

In this section, the LED example from Section 2 is used to illustrate the application of the proposed method. Based on the stress setting conditions for the absolute ambient temperature and driven current presented in Tsai et al. [18], we take \(k = 6\), \(p_0 = 0\), \(q_i = 10\), for \(i = 1, 2, 3, 4\); \(p_5 = 1\), \(q_5 = 6\) and \(p_6 = 2\), \(q_6 = 8\). All stress loading levels are taken as the settings given in Section 2. Moreover, the length of the measuring time interval is taken as \(\delta = 1008\) hours, and the ADT will be ended at the 6048th hour. The scheduled measuring times will then be in the interval \([1008, 6048]\). To implement the searching procedure provided in Section 3, the interval \([1008, 6048]\) is divided into 5 subintervals of length 1008. Hence, the partition points are 1008, 2000, 3024, 4032, 5040 and 6048. It follows that \(B_j = \{t | 1008 \leq t_1, \ldots, t_6 \leq 6048 \text{ and } TC < \phi_0\}\).

It is assumed that a LED is classified as failure if 30% luminous flux is lost from the initial amount. The parameters, \(\gamma_0\), \(\gamma_1\), \(\gamma_2\), \(\gamma_3\) and \(\beta\) in the lifetime distribution are replaced by their MLEs, \(\hat{\gamma}_0\), \(\hat{\gamma}_1\), \(\hat{\gamma}_2\), \(\hat{\gamma}_3\) and \(\hat{\beta}\), respectively. In addition, the costs of experimentation are taken as \(c_{L_2} = 0.001\), \(c_{L_3} = d_i \times c_{L_2}\), \(d_i = 5, 10\), \(c_0 = 50\), \(c_{op} = 0.6\) with lot sizes \(N_d = 1000, 5000\), experiment budgets \(\phi_0 = 10000, 20000\), and the reference lifetime percentiles of \(p = 0.1\) and \(p = 0.5\) for the ADT, respectively. Optimal ADT strategies are obtained and given in Table 1 via the algorithm proposed in Section 3. From Table 1, it can be seen that \(\tilde{\sigma}(n, t^*)\) is decreased if a larger experiment budget is available for the ADT. Hence, the optimal ADT strategy would be established so that a total cost is close to the experiment budget if it is possible. Table 1 shows that increasing the sample size in each experiment run could decrease the value of \(\tilde{\sigma}(n^*, t^*)\) if more experiment budget is allowed. The planning ADT inference provides a more accurate estimate of the
smaller $100p^\text{th}$ percentile than the estimate of the larger $100p^\text{th}$ percentile in terms of the $\sigma(n^*, t^*)$. It also needs smaller total cost to estimate a larger $100p^\text{th}$ percentile than the total cost to estimate a smaller $100p^\text{th}$ percentile. But the difference of total costs for estimating these two percentiles, with $p = 0.1$ and 0.5, is insignificant. Finally, Table 1 reveals that the total cost with $d_1 = 5$ is slightly larger than the total cost with $d_1 = 10$ under the condition of $\phi_0 = 10000$. But the total cost with $d_1 = 5$ is little bit smaller than that with $d_1 = 10$ under the budget condition of $\phi_0 = 20000$. However, the difference between two total costs for different values of $d_1$ is insignificant. Hence, the effect of increasing the variable cost on unit absolute ambient temperature 5 times or higher over the variable cost on unit driven current is insignificant.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$N_D$</th>
<th>$d_1$</th>
<th>$\phi_0$</th>
<th>$n^*$</th>
<th>$t_1^*$</th>
<th>$t_2^*$</th>
<th>$t_3^*$</th>
<th>$t_4^*$</th>
<th>$t_5^*$</th>
<th>$t_6^*$</th>
<th>TC</th>
<th>$\sigma(n^<em>, t^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1000</td>
<td>5</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>3024</td>
<td>2016</td>
<td>5040</td>
<td>1008</td>
<td>1008</td>
<td>9998.74</td>
<td>161.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>5040</td>
<td>2016</td>
<td>2016</td>
<td>1008</td>
<td>1008</td>
<td>19989.66</td>
<td>112.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>4032</td>
<td>2016</td>
<td>2016</td>
<td>1008</td>
<td>1008</td>
<td>9979.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>1008</td>
<td>1008</td>
<td>19998.44</td>
<td>112.58</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>5</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>3024</td>
<td>2016</td>
<td>5040</td>
<td>1008</td>
<td>1008</td>
<td>9998.74</td>
<td>161.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>5040</td>
<td>2016</td>
<td>2016</td>
<td>1008</td>
<td>1008</td>
<td>19989.66</td>
<td>112.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>4032</td>
<td>2016</td>
<td>2016</td>
<td>1008</td>
<td>1008</td>
<td>9979.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>1008</td>
<td>1008</td>
<td>19998.44</td>
<td>112.58</td>
</tr>
<tr>
<td>0.5</td>
<td>1000</td>
<td>5</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>2016</td>
<td>3024</td>
<td>3024</td>
<td>9927.23</td>
<td>361.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>2016</td>
<td>1008</td>
<td>4032</td>
<td>1008</td>
<td>2016</td>
<td>19956.62</td>
<td>251.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>3024</td>
<td>2016</td>
<td>1008</td>
<td>9913.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>2016</td>
<td>1008</td>
<td>2016</td>
<td>1008</td>
<td>2016</td>
<td>19983.79</td>
<td>251.88</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>5</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>2016</td>
<td>3024</td>
<td>3024</td>
<td>9927.23</td>
<td>361.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>2016</td>
<td>1008</td>
<td>4032</td>
<td>1008</td>
<td>2016</td>
<td>19956.62</td>
<td>251.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>10000</td>
<td>31</td>
<td>6048</td>
<td>1008</td>
<td>1008</td>
<td>3024</td>
<td>2016</td>
<td>1008</td>
<td>9913.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20000</td>
<td>64</td>
<td>6048</td>
<td>2016</td>
<td>1008</td>
<td>2016</td>
<td>1008</td>
<td>2016</td>
<td>19983.79</td>
<td>251.88</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, a planning strategy is suggested to implement a two-variable accelerated degradation test for highly reliable products. A total cost function, which consists of the fixed cost, operating cost and variable cost, is used to establish the optimum accelerated degradation test plans in order to minimize the asymptotic variance of the $100p^\text{th}$ lifetime percentile estimator at the normal use condition, given experiment budget, under Wiener process.

The lumen degradation data set of light emitting diodes is used to illustrate the proposed method. Optimal strategies under different combinations of cost components and two percentiles of concern have been constructed and summarized in Table 1 for reference. The proposed method can provide a reasonable planning strategy for the accelerated degradation testing procedure proposed by Tasi et al. [18].

Warranties are important for marketing highly reliable products. Good warranty policies make the products more competitive in the market. Considering a warranty cost for the planning strategy of the accelerated degradation testing experiment is an important issue for product providers. Next, a pitfall of using a Wiener process is the possibility of getting a negative cumulative damage. Authors are currently working on further research works of planning strategy under different stochastic processes with warranties.
Acknowledgements

Authors would like to thank the editor, associate editor and anonymous referees for their suggestions and comments that led to a significant improvement of this manuscript. Dr. T.-R. Tsai's research is supported by the grant of National Science Council, Taiwan NSC 100-2221-E-032-014.

References


**Appendix**

Since \( E(s_y) = \mu_L = C / \nu_L \), \( E(\Delta x_{y|h}) = \nu_L \Delta \tau_{y|h} \), \( E(C - 2\nu_L s_y) = -C \) and \( E(\Delta x_{y|h} - 2\nu_L \Delta \tau_{y|h}) = -\nu_L \Delta \tau_{y|h} \). We obtain the following results:

\[
a_1 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_0} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L; \\
a_2 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_0 \partial \gamma_1} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L; \\
a_3 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_0 \partial \gamma_2} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L; \\
\]

\[a_4 = a_4; a_5 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_3} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L \nu_L; \]

\[a_6 = a_4; a_7 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_3} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L \nu_L; \]

\[a_8 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_3} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L \nu_L; \]

\[a_9 = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_3} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L \nu_L; \]

\[a_{10} = -E\left( \frac{\partial^2 f(\Theta)}{\partial \gamma_3} \right) = \frac{1}{\beta^2} \sum_{i=1}^{k} \left( \int \frac{p_i C + \sum_{j=1}^{m_i} \nu_L \Delta \tau_{j|h}}{\nu_L} \right) \nu_L \nu_L \nu_L \nu_L; \]
\[ a_{11} = -E\left( \frac{\partial^2 \ell(\Theta)}{\partial \beta^2} \right) = \frac{-N}{\beta^2} + 3 \frac{k}{\beta^4} \sum_{j=1}^k \left\{ \frac{p_j}{\beta^2} \sum_{i=1}^k \left[ \frac{(C - v_i s_i)^2}{s_i} \right] + \frac{q_i}{\beta^2} \sum_{j=1}^k m_{ij} \beta^2 \right\} \]

\[ -\frac{1}{\beta^2} \sum_{j=1}^k \sum_{j=1}^q \sum_{i=1}^m E \left[ \frac{4 y_{ijh}}{e^{2y_{ijh}/\beta^2}} - 1 \right] \left( 3 - \frac{4 y_{ijh}}{\beta^2} \frac{e^{2y_{ijh}/\beta^2}}{e^{2y_{ijh}/\beta^2} - 1} \right) \].

Authors’ Biographies:

**Tzong-Ru Tsai** is currently a Professor in the Department of Statistics at Tamkang University, Taiwan. He received his Ph.D. in Statistics in 1996 from National Chengchi University, Taiwan. His major research interests are in quality control and reliability analysis.

**Yuhlong Lio** is a Professor in the Department of Mathematical Sciences at the University of South Dakota, USA. His research interests include reliability, smooth estimator, survival analysis and statistical process control. He received his B.S. in Mathematics from National Cheng-Kung University, his M.S. in Mathematics from National Central University and his Ph.D. in statistics from the University of South Carolina in 1987.

**Nan Jiang** is an Associate Professor in the Department of Mathematical Sciences at the University of South Dakota, USA. She received her Ph.D. in Mathematics from Kansas State University in 2000. Her research interests include the entropy convergence of finite difference schemes for conservation laws, reliability and statistical quality control.