Reliability Characteristics of a Server System with Asynchronous and Synchronous Replications

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Abstract: Recently, replication mechanisms for a disaster recovery have been widely used for the server system. This paper considers the reliability in server systems with synchronous and asynchronous replications for the disaster recovery. We formulate two stochastic models of server systems with synchronous and with asynchronous replications: In synchronous replication, the server transmits the database content to a backup site as soon as a main site updates the storage database. In asynchronous replication, the server transmits the database content to a backup site after a constant time. We derive the expected number of the replication and the probability of migrating routine work to the backup site and the probability of the system down. Further, in asynchronous model, we calculate the cost effectiveness and discuss an optimal policy to minimize it. Finally, in asynchronous and in synchronous models, numerical examples are given to compare the performance of them.

Keywords: Asynchronous, cost effectiveness, disaster recovery, replication, synchronous.

1. Introduction

As the network technology has remarkably developed, a backup copy of database in a main site has been accumulated in a remote server. That is, the backup copy has been transmitted to the remote server using a network link in order to avoid the data loss due to a disaster such as fire disaster, electricity failure, earthquake and typhoon and so on. In this way, replication mechanisms for a disaster recovery have been widely used for a server system: The system provides the backup site with the storage for client's backup copies as well as the server for business restart. When a disaster occurs in a main site, the server system migrates the routine work from the main site to a backup site. The server in the main site transmits the database content from the main site to the backup site using a network link. This is called replication [1].

There are two ways of the replication which are synchronous and asynchronous schemes: In the synchronous scheme, the server in a main site transmits the database content from a main site to a backup site as soon as the server updates the storage database when a client requests the data update. In the asynchronous scheme, after the server in a main site updates the storage database, the server transmits the database content to a backup site at any time [1]. Synchronous scheme guarantees consistency of database content but has a prohibitive cost. Asynchronous scheme has lower costs than it but can compromise consistency. These schemes have been investigated from various view points. In references [2] and [3], they have discussed the problems to improve system performance such as recovery time objective, and recovery point objective. It is also important to improve reliability of the server system with the replication in order to restore a consistent state of
database. For example, when the interval of request for the data update is large, we should apply synchronous replication rather than asynchronous replication because replication costs are very small.

This paper considers the reliability of server systems with asynchronous and synchronous replications. We formulate two stochastic models of server systems with synchronous and with asynchronous replications: The server in the main site updates the storage database when a client requests the data update. Then, in synchronous replication, the server transmits the database content to a backup site as soon as a main site updates the storage database. In asynchronous replication, the server transmits the database content to a backup site after a constant time. We derive the expected number of replications, the probability of migrating routine work to the backup site and the probability of the system down. Further, in asynchronous model, we calculate the cost effectiveness and discuss an optimal replication interval to minimize it. Finally, in asynchronous and in synchronous models, numerical examples are given to compare the performance of them.

2. Model and Analysis

A server system consists of a monitor, a main site and a backup site as shown in Figure 1: Both main and backup sites consist of identical servers and storages. The backup site stands by the alert when a disaster has occurred. The server in the main site performs the routine work and updates the storage database when a client requests the data update. The monitor can replicate the database content from the main site to the backup site only when the main site is in server idling and the backup site is in a normal condition.

![Figure 1. Outline of a server system.](image)

2.1. Synchronous Scheme

The operation of a server system with synchronous scheme is shown in Figure 2.

1. A disaster occurs in the main site according to an exponential distribution \((1 - e^{-\lambda t})\), and the server becomes unavailable.

2. A client requests the data update to the storage. The request time has an exponential distribution \((1 - e^{-\alpha t})\) and the update time has an exponential distribution \((1 - e^{-\beta t})\).
(3) The server transmits the database content to a backup site as soon as a main site updates the storage database.

(a) If the backup site is available, the monitor orders the server to transmit the database content from the main site to the backup site (replication). The replication time has an exponential distribution \((1 - e^{-\lambda t})\).

(i) A disaster occurs in the backup site according to an exponential distribution \((1 - e^{-\lambda t})\). If a disaster has occurred in either of main site or backup site while it replicates the database content to the backup site, the server system becomes faulty (system down).

(ii) If the backup site is in disaster, the monitor waits its recovery, and thereafter, replicates the database content to the backup site. The recovery time of the backup site has an exponential distribution \((1 - e^{-\lambda t})\).

(b) If the main site is in server down and the backup site is available, the routine work is migrated from the main site to the backup site (system migration). If the backup site is in a disaster, the server system becomes faulty (system down).

We define the following states of the backup site:

State 0: Backup site is in a normal condition.

State 1: Disaster occurs.

The states of the backup site defined above form a two-state Markov process [5]. We have the following probabilities \(P_i(t)\) that the backup site is in state \(i(i = 0)\) at time \(t\) and state \(j(f = 0,1)\) at time \(t(>0)\) [4].

\[
P_{00}(t) = \frac{\gamma}{\lambda_2 + \gamma} + \frac{\lambda_2}{\lambda_2 + \gamma} e^{-(\lambda_2 + \gamma)t}, \quad P_{01}(t) = 1 - P_{00}(t).
\]

Under the above assumptions, we define the following states of the server system:

State 5: System begins to operate or restart.

State 6: Main site is in data update and backup site is in a normal condition.

State 7: Main site has updated the storage database and backup site is in disaster.

State 8: While main site is in data update, backup site is in disaster.

Figure 2. Outline of a synchronous model.
State \(R\): Replication begins.
State \(F\): System is down.
State \(SW\): Disaster occurs in main site and routine work is migrated to backup site.

The system states defined above form a Markov renewal process, where \(F\) and \(SW\) are absorbing states. A transition diagram between system states is shown in Figure 3.

\[
\begin{align*}
q_{5,6}(s) &= \frac{\alpha(s + \lambda_1 + \alpha + \gamma)}{A(s)}, \\
q_{5,8}(s) &= \frac{\alpha \lambda_2}{A(s)}, \\
q_{5,SW}(s) &= \frac{\lambda_1(s + \lambda_1 + \alpha + \gamma)}{A(s)}, \\
q_{5,F}(s) &= \frac{\lambda_1 \lambda_2}{A(s)}, \\
q_{6,R}(s) &= \frac{\beta}{s + \lambda_1 + \lambda_2 + \beta}, \\
q_{6,8}(s) &= \frac{\lambda_2}{s + \lambda_1 + \lambda_2 + \beta}, \\
q_{6,SW}(s) &= \frac{\lambda_1}{s + \lambda_1 + \lambda_2 + \beta}, \\
q_{7,8}(s) &= \frac{\alpha}{s + \lambda_1 + \alpha + \gamma}, \\
q_{7,R}(s) &= \frac{\gamma}{s + \lambda_1 + \alpha + \gamma}, \\
q_{7,F}(s) &= \frac{\lambda_1}{s + \lambda_1 + \alpha + \gamma}, \\
q_{8,6}(s) &= \frac{\gamma}{s + \lambda_1 + \beta + \gamma}, \\
q_{8,7}(s) &= \frac{\beta}{s + \lambda_1 + \beta + \gamma}, \\
q_{8,F}(s) &= \frac{\lambda_1}{s + \lambda_1 + \beta + \gamma}, \\
q_{R,5}(s) &= \frac{\omega}{s + \lambda_1 + \lambda_2 + \omega}, \\
q_{R,F}(s) &= \frac{\lambda_1 + \lambda_2}{s + \lambda_1 + \lambda_2 + \omega}, \\
\end{align*}
\]

where \(A(s) = (s + \lambda_1 + \alpha)(s + \lambda_1 + \lambda_2 + \alpha + \gamma)\).

Figure 3. Transition diagram between a server system states.
We derive the probability $P_F$ of the system down and the probability $P_{SW}$ of migrating the routine work to the backup site. The probability distributions $P_{i,F}(t)$ ($i = 5, 6, 7, 8, R$) from state $i$ to state $F$ until time $t$ are

$$P_{5,F}(t) = Q_{5,F}(t) + Q_{5,6}(t) \cdot P_{5,F}(t) + Q_{5,8}(t) \cdot P_{8,F}(t),$$

$$P_{6,F}(t) = Q_{6,8}(t) \cdot P_{8,F}(t) + Q_{6,R}(t) \cdot P_{R,F}(t),$$

$$P_{7,F}(t) = Q_{7,F}(t) + Q_{7,8}(t) \cdot P_{8,F}(t) + Q_{7,R}(t) \cdot P_{R,F}(t),$$

$$P_{8,F}(t) = Q_{8,F}(t) + Q_{8,6}(t) \cdot P_{6,F}(t) + Q_{8,7}(t) \cdot P_{7,F}(t),$$

$$P_{R,F}(t) = Q_{R,F}(t) + Q_{R,5}(t) \cdot P_{5,F}(t).$$

Taking the LS transforms of (1)-(5) and arranging them,

$$P_{5,F}(s) = \frac{q_{5,F}(s) + x_1(s)[q_{8,F}(s) + q_{8,7}(s)q_{7,F}(s)] + y_1(s)q_{R,F}(s)}{1 - y_1(s)q_{R,5}(s)},$$

where

$$x_1(s) = \frac{q_{5,6}(s)q_{6,8}(s) + q_{s,8}(s)}{1 - q_{8,6}(s)q_{6,8}(s) - q_{8,7}(s)q_{7,8}(s)},$$

$$y_1(s) = q_{5,6}(s)q_{6,R}(s) + x_1(s)[q_{8,6}(s)q_{6,R}(s) + q_{8,7}(s)q_{7,R}(s)].$$

Hence, the probability $P_F$ of the system down is,

$$P_F = \lim_{s \to 0} P_{5,F}(s) = \frac{q_{5,F}(0) + x_1(0)[q_{8,F}(0) + q_{8,7}(0)q_{7,F}(0)] + y_1(0)q_{R,F}(0)}{1 - y_1(0)q_{R,5}(0)}.$$

Similarly, the probability $P_{SW}$ of migrating the routine work to the backup site is given as follows:

$$P_{SW} = \frac{q_{5,SW}(0) + [q_{5,6}(0) + x_1(0)q_{8,6}(0)]q_{6,SW}(0)}{1 - y_1(0)q_{R,5}(0)}.$$

It is evident that $P_F + P_{SW} = 1$.

### 2.2. Asynchronous Scheme

The operation of a server system with asynchronous scheme is shown in Figure 4.

1. The monitor confirms instantly the state of the main site at constant time $T$, where $V(t) = 0$ for $t < T$ and $1$ for $t \geq T$. The main site is in one of three states: server idling, data update and server down.

   a. If the main site is in server idling and the backup site is available, the monitor orders the server to transmit the database content from the main site to the backup site (replication).

   i. If a disaster has occurred in either of main site or backup site while it replicates the database content to the backup site, the server system becomes faulty (system down).
(b) If the main site is in server idling and the backup is in disaster, the monitor waits its recovery, and thereafter, replicates the database content to the backup site.

Figure 4. Outline of an asynchronous model.

(c) If the main site is in data update, the monitor waits until the data update is completed, and thereafter, replicates the database content from the main site to the backup site.

We define the following states of main site:

State 2: Idling begins.

State 3: Data update begins.

State 4: Disaster occurs.

Similarly, we have the following probabilities $P_j(i=2)$ at time $0$ and state $j(j=2,3,4)$ at time $t(>0)$ [4].

$$P_{22}(t) = \frac{\alpha}{\alpha + \beta} e^{-(\alpha + \beta + \lambda)t} + \frac{\beta}{\alpha + \beta} e^{-\lambda t},$$

$$P_{23}(t) = \frac{\alpha}{\alpha + \beta} e^{-\lambda t} - \frac{\alpha}{\alpha + \beta} e^{-(\alpha + \beta + \lambda)t},$$

$$P_{24}(t) = 1 - e^{-\lambda t}.$$

Under the same assumptions as Section 2.1, we define the following states of the server system:

State 5: System begins to operate or restart.

State 6: When the monitor confirms the state of main site at time $T$, it is in data update and the backup site is in a normal condition.

State 7: When the monitor confirms the state of main site at time $T$, it is in server idling and the backup site is in disaster.

State 8: When the monitor confirms the state of main site at time $T$, it is in data update and the backup site is in disaster.
(b) If the main site is in server idling and the backup is in disaster, the monitor waits its recovery, and thereafter, replicates the database content to the backup site.

(c) If the main site is in data update, the monitor waits until the data update is completed, and thereafter, replicates the database content from the main site to the backup site.

We define the following states of main site:

- State 2: Idling begins.
- State 3: Data update begins.
- State 4: Disaster occurs.

Similarly, we have the following probabilities \( P_{ij} \) that the main site is in state \( i \) at time 0 and state \( j \) at time \( t \).

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Under the same assumptions as Section 2.1, we define the following states of the server system:

- State 5: System begins to operate or restart.
- State 6: When the monitor confirms the state of main site at time \( T \), it is in data update and the backup site is in a normal condition.
- State 7: When the monitor confirms the state of main site at time \( T \), it is in server idling and the backup site is in disaster.
- State 8: When the monitor confirms the state of main site at time \( T \), it is in data update and the backup site is in disaster.

State \( R \): Replication begins.

State \( F \): System is down.

State \( S_{w} \): Disaster occurs in main site and routine work is migrated to backup site.

The system states defined above form a Markov renewal process, where \( F \) and \( S_{w} \) are absorbing states. A transition diagram between system states is shown in Figure 5.

The LS transforms of transition probabilities \( Q_{i,j}(t)(i = 5, 6, 7, 8, R; j = 5, 6, 7, 8, R, F, S_{w}) \) are given by the following equations:

\[
q_{5,R}(s) = e^{-\gamma T} P_{22}(T) P_{00}(T),
q_{5,6}(s) = e^{-\gamma T} P_{23}(T) P_{00}(T),
q_{5,7}(s) = e^{-\gamma T} P_{22}(T) P_{01}(T),
q_{5,8}(s) = e^{-\gamma T} P_{23}(T) P_{01}(T),
q_{5,S_{w}}(s) = e^{-\gamma T} P_{24}(T) P_{01}(T),
q_{5,F}(s) = e^{-\gamma T} P_{24}(T) P_{01}(T),
q_{6,R}(s) = \frac{\lambda}{s + \lambda_1 + \lambda_2 + \gamma},
q_{6,5}(s) = \frac{\lambda_2}{s + \lambda_1 + \lambda_2 + \gamma},
q_{6,S_{w}}(s) = \frac{\lambda_1}{s + \lambda_1 + \lambda_2 + \gamma},
q_{6,F}(s) = \frac{\gamma}{s + \lambda_1 + \alpha + \gamma},
q_{7,R}(s) = \frac{\alpha}{s + \lambda_1 + \alpha + \gamma},
q_{7,F}(s) = \frac{\lambda_1}{s + \lambda_1 + \alpha + \gamma},
q_{7,S_{w}}(s) = \frac{\gamma}{s + \lambda_1 + \beta + \gamma},
q_{7,F}(s) = \frac{\beta}{s + \lambda_1 + \beta + \gamma},
q_{7,F}(s) = \frac{\lambda_1}{s + \lambda_1 + \beta + \gamma},
q_{7,F}(s) = \frac{\omega}{s + \lambda_1 + \lambda_2 + \omega},
q_{7,F}(s) = \frac{\lambda_1 + \lambda_2}{s + \lambda_1 + \lambda_2 + \omega}.
\]

First, we derive the expected number \( M_{R}(T) \) of the replication. The LS transforms \( m_{i,R}(s) \) of the expected number distributions \( M_{i,R}(t)(i = 5, 6, 7, 8) \) from state \( i \) to state \( R \)
until time $t$ are
\[
m_{5,R}(s) = q_{5,R}(s) + q_{5,R}(s)q_{R,5}(s)m_{5,R}(s) + q_{5,6}(s)m_{6,R}(s)
q_{5,7}(s)m_{7,R}(s) + q_{5,8}(s)m_{8,R}(s),
\]
\[
m_{6,R}(s) = q_{6,R}(s) + q_{6,R}(s)q_{R,5}(s)m_{5,R}(s) + q_{6,8}(s)m_{8,R}(s),
\]
\[
m_{7,R}(s) = q_{7,R}(s) + q_{7,R}(s)q_{R,5}(s)m_{5,R}(s) + q_{7,8}(s)m_{8,R}(s),
\]
\[
m_{8,R}(s) = q_{8,6}(s)m_{6,R}(s) + q_{8,7}(s)m_{7,R}(s).
\]

Hence, the expected number $M_R(T)$ until either system down or system migration is
\[
M_R(T) = \lim_{s \to 0} m_{5,R}(s) = \frac{Y(T)}{1 - Y(T)q_{R,5}(0)},
\]

where
\[
X(T) = \frac{P_{23}(T)P_{00}(T)q_{6,8}(0) + P_{22}(T)P_{01}(T)q_{7,8}(0) + P_{23}(T)P_{01}(T)}{1 - q_{6,8}(0)q_{6,8}(0) - q_{7,8}(0)}
\]
\[
Y(T) = P_{22}(T)P_{00}(T) + P_{23}(T)P_{00}(T)q_{6,R}(0) + P_{22}(T)P_{01}(T)q_{7,R}(0)
+ X(T)[q_{6,6}(0)q_{6,6}(0) + q_{7,7}(0)q_{7,7}(0)].
\]

Similarly, the expected number $M_S(T)$ of monitoring the main site, the probability $P_F(T)$ of the system down and the probability $P_{S_u}(T)$ of migrating the routine work to the backup site are given as follows:
\[
M_S(T) = q_{R,5}(0)M_R(T),
\]
\[
P_F(T) = \frac{P_{24}(T)P_{01}(T) + P_{22}(T)P_{01}(T)q_{7,F}(0)}{1 - q_{7,F}(0)q_{7,F}(0)}
+ X(T)[q_{7,F}(0)q_{7,F}(0) + Y(T)q_{R,F}(0)]
\]
\[
P_{S_u}(T) = 1 - P_F(T).
\]

### 2.3. Optimal Policy

In this section, when we apply an asynchronous scheme to the server system, we calculate the cost effectiveness and derive an optimal replication interval $T^*$ to minimize it. Let $c_S$ be the cost for monitoring the main site, $c_R$ be the cost for the replication and $c_F$ be the cost for the system down. Then, we define $P_{S_u}(T)$ as effectiveness, and define the cost effectiveness $E(T)$ as follows:
\[
E(T) = \frac{c_S M_S(T) + c_R M_R(T) + c_F P_F(T)}{P_{S_u}(T)}.
\]

We seek an optimal replication interval $T^*$ which minimizes $E(T)$ in (17). It is clearly seen that $\lim_{T \to 0} E(T) = \infty$ and $\lim_{T \to \infty} E(T) = c_F \lambda_2 / \gamma$. There exists a finite $0 < T^* \leq \infty$. From $dE(T)/dT = 0$,
\[
\frac{M'_R(T)P_{S_R}(T)}{P'_{S_R}(T)} - M_R(T) = \frac{c_F}{c_Sq_{RS}(0) + c_R},
\]
where \( \Phi'(t) = d\Phi(t)/dt \). Denoting the left-hand side of (18) by \( L(T) \),
\[
\frac{dL(T)}{dT} = \left( \frac{M'_R(T)}{P'_{S_R}(T)} \right) P_{S_R}(T).
\]

When \( M'_R(T)/P'_{S_R}(T) \) is strictly increasing in \( T \), \( L(T) \) is also strictly increasing in \( T \) and \( \lim_{T \to 0} L(T) = -1/(1 - q_{RS}(0)) \). Therefore, we have the following optimal policy:

(i) If \( L(\infty) > c_F /[c_Sq_{RS}(0) + c_R] \) then there exists a finite and unique \( T^*(< \infty) \) which satisfies (18).

(ii) If \( L(\infty) \leq c_F /[c_Sq_{RS}(0) + c_R] \) then \( T^* = \infty \).

3. Numerical Examples

We compute numerically an optimal replication interval \( T^* \) which minimizes \( E(T) \) in (17). Suppose that the mean time \( 1/\beta \) required for the data update is a unit time. It is assumed that the mean generation interval of request for the data update is \( (1/\alpha)/(1/\beta) = 10 - 50 \), the mean generation interval of a disaster is \( (1/\lambda_1)/(1/\beta) = (1/\gamma)/(1/\beta) = 200, 400 \), the mean time required for recovery of the backup site is \( 1/\gamma = 90 - 240 \), the mean time required for the replication is \( 1/w \), and \( 1/\beta = 1, 1/\alpha = 10, 1/w = 1, 1/\lambda_1 = 1/\lambda_2 = 200, 1/\gamma = 90 \). Further, the respective costs for monitoring the main site, for the replication and for the system down are \( c_S = 1, c_R = 2, c_F = 500 \) and \( c_F / c_S = 500, 1000 \).

Table 1 gives the optimal replication interval \( T^* \) which minimizes the cost effectiveness \( E(T) \). For example, when \( c_S = 1, c_R = 2, c_F = 500 \) and \( 1/\beta = 1, 1/\alpha = 10, 1/w = 1, 1/\lambda_1 = 1/\lambda_2 = 200, 1/\gamma = 90 \), \( T^* \) is 40.6. This indicates that \( T^* \) increase with \( 1/w \) and \( 1/\alpha \). Further, \( T^* \) decrease with \( 1/\gamma \). In this case, we should monitor both main and backup sites frequently. Moreover, \( T^* \) increase with \( 1/\lambda_1 \) or \( 1/\lambda_2 \) and decrease with \( c_F / c_S \).

<table>
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<tr>
<th>( c_F / c_S )</th>
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<th>( 1/w )</th>
<th>( 1/\gamma )</th>
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Figures 6 and 7 give probabilities $P_{Sw}$ in synchronous and $P_{Sw}(T^*)$ in asynchronous of migrating routine work to the backup site. Figure 6 indicates that $P_{Sw}(T^*)$ is greater than $P_{Sw}$ when $1/\alpha$ is less than $1/\alpha^*(=19.0)$, but when $1/\alpha$ is greater than $1/\alpha^*$, $P_{Sw}$ is greater than $P_{Sw}(T^*)$. That is, when $1/\alpha$ is large, we should apply synchronous replication in the server system. On the other hand, Figure 7 indicates that both $P_{Sw}(T^*)$ and $P_{Sw}$ decrease with $1/\gamma$, and $P_{Sw}(T^*)$ is greater than $P_{Sw}$. Thus, we should apply asynchronous replication to the server system.

![Figure 6](image)

**Figure 6.** $P_{Sw}$ when $1/w = 2, 1/\lambda_1 = 1/\lambda_2 = 200, 1/\gamma = 90, c_R / c_5 = 2, c_F / c_5 = 500$.

![Figure 7](image)

**Figure 7.** $P_{Sw}$ when $1/\alpha = 10, 1/\omega = 2, 1/\lambda_1 = 1/\lambda_2 = 200, c_R / c_5 = 2, c_F / c_5 = 500$. 
### 4. Conclusions

We have considered the problem of reliability in server systems with asynchronous replication and synchronous replication for the disaster recovery. We have formulated two stochastic models of server systems with synchronous and with asynchronous replications, and derived the expected number of the replication and the monitoring, the probabilities of migrating routine work to the backup site and of the system down. Further, in asynchronous replication, we have calculated the cost effectiveness and discuss the optimal replication interval to minimize it.

From numerical examples, we have shown that the optimal replication interval increases with interval of request for the data update and decrease with the time for the recovery of the backup site. On the other hand, we have shown that when the interval of request is greater than a constant time, we should apply synchronous replication rather than asynchronous one in the server system.

Further, it would be important to evaluate and improve the reliability of a server system with replication schemes. Such study would be expected to apply to actual fields.

### References


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