An Adjustable Channel Bonding Strategy in Centralized Cognitive Radio Networks and its Performance Optimization

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Abstract: One form of spectrum enhancement technologies for cognitive radio networks is a channel bonding strategy, with which a network user can utilize multiple idle channels as one bonding channel. In this paper, we propose an adjustable channel bonding strategy for centralized cognitive radio networks. The main idea is that the number of channels to be aggregated is set according to the number of packets remaining in the system dynamically. Based on the working principle of the proposed adjustable channel bonding strategy, we present a discrete-time priority queueing model with an adjustable transmission rate for this network system. By constructing a two-dimensional Markov chain, we give the transition probability matrix of the Markov chain and obtain the steady-state distribution of the system model. Accordingly, we derive the formulas for the blocking rate, the normalized throughput, the average latency of the Secondary Users (SUs) and the closed channel ratio. Moreover, we provide numerical experiments to investigate the influence of the buffer capacity of the SUs on different performance measures. Finally, with regard to the joining of an SU packet as a trial, we give the Nash equilibrium strategy and socially optimal strategy for the SUs, and propose a reasonable admission fee in order to oblige the SU packets to adopt the socially optimal strategy.

Keywords: Adjustable transmission rate, channel bonding, cognitive radio networks, Markov chain, optimal strategy, priority queue.

1. Introduction

Radio spectrum, due to its scarcity, is the most invaluable resource of wireless networks. How to reasonably allocate the spectrum to network users has been the focus of many recent studies. In order to better utilize radio spectrum, the Federal Communication Committee (FCC) has recently suggested a new policy for dynamically allocating spectrum, which is the basis for the creation of cognitive radio networks. According to the dynamic spectrum allocation strategy, there are two types of users in cognitive radio networks; namely, Primary Users (PUs) and Secondary Users (SUs). PUs refer to the licensed users who have priority in occupying the spectrum. SUs refer to the network users who can make opportunistic use of the spectrum when the spectrum is not occupied by the PUs [9].

The dynamic spectrum allocation strategy in cognitive radio networks can effectively eliminate the spectrum hole and greatly improve the utilization of the network spectrum [3].

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In cognitive radio networks, the dynamic spectrum allocation strategies can be classified into distributed protocols and centralized protocols.

With the distributed protocols, the SUs will perform spectrum sensing and spectrum scheduling individually, and there is no central controller in the system. Zhang and Su [11] proposed and analyzed a medium access control protocol with cooperative sequential spectrum sensing scheme for the wireless cognitive radio networks. By applying the $M/G^V/1$ queueing model, they evaluated the performance of the proposed protocol for both the saturation network case and non-saturation network case.

With the centralized protocols, the spectrum allocation is controlled by a central controller, and the central controller can allocate the spectrum for the network users [8]. Kaur et al. [5] considered a centralized cognitive radio network where the central controller could coordinate the spectrum allocation with the surrounding cognitive radio in the network. They developed an analytical framework for evaluating the performance of the SUs by studying the queueing behavior of the packets in the system. Generally speaking, due to the influence of the central controller, the throughput of centralized networks is better than that of distributed networks, especially when there is a heavy traffic load.

In cognitive radio network research, the channel bonding strategy is one form of spectrum enhancement technology where available channels can be aggregated into one channel. Several studies have focused on channel bonding strategy. Zhao et al. [12] proposed a dynamic channel bonding strategy and built a discrete-time priority queueing model in order to derive the throughput and the average latency for the SUs in cognitive radio networks. Konishi et al. [6] built a continuous-time priority queueing model and investigated the impact of the number of sub-channels to be aggregated on the system performance in cognitive radio networks. They assumed that a fixed number of sub-channels were aggregated into one channel for an SU under a non-contiguous channel bonding scheme.

A few studies have been done focusing on how to control the queue in cognitive radio networks [1]. Li and Han [7] discussed the individually and socially optimal strategies in cognitive radio networks. They assumed the queue length can be observed by the SUs before making decisions on whether or not to join the queue. Additionally, they proposed a pricing scheme to coincide the two strategies. Do et al. [2] studied a non-cooperative game problem for queueing control in cognitive radio network. In contrast to [7], they assumed the SUs did not have any information on the queue length before making decisions. They applied an $M/M/1$ queueing model, and discussed the individual equilibrium strategy and social optimization for the SUs.

Conclusively, we note that in most researches on the channel bonding strategies in cognitive radio networks, the number of aggregated channels was assumed to be fixed. Obviously, this kind of static bonding strategy cannot dynamically adapt the varying environments and conditions. We also find that most of the studies on the performance of channel bonding strategy were carried out in a continuous-time field, and the transmission of the SU packets was assumed to be guaranteed so long as they join the system. It is well known that modern communications are often digital, discrete-time models are more appropriate for designing and analyzing networks. On the other hand, in practical networks, the SU packets may be blocked or dropped. It means that the transmission of SU packets will not be guaranteed.

In this paper, in order to make the channel bonding strategy more flexible, we propose an adjustable channel bonding strategy, in which the number of the channels to be...
aggregated is dependent on the number of packets in the system. Considering the digital nature of modern communications, we present a discrete-time queueing model to evaluate the system performance of the proposed adjustable channel bonding strategy. By constructing a two-dimensional Markov chain, we obtain the steady-state distribution of the system model. Accordingly, we derive the formulas for the blocking rate, the normalized throughput, the average latency of the SUs and the closed channel ratio. We also provide numerical experiments to show the influence of the buffer capacity of the SUs on different performance measures. Finally, we give Nash equilibrium strategy and socially optimal strategy by considering the failed transmission of the SU packets, then we propose a reasonable admission fee for SU packets to oblige the coincidence of the two strategies.

The rest of this paper is organized as follows: In Section 2, an adjustable channel bonding strategy in centralized cognitive radio networks is proposed, and the system model is built. In Section 3, the performance analysis by using a two-dimensional Markov chain is presented. In Section 4, the formulas for the performance measures in terms of the blocking rate, the normalized throughput, the average latency of the SUs and the closed channel ratio are given. Numerical experiments are also provided to evaluate the system performance. In Section 5, optimization of the system performances for the SUs is given. Finally, conclusions are drawn in Section 6.

2. System Model for an Adjustable Channel Bonding Strategy

2.1. An Adjustable Channel Bonding Strategy

In this paper, we consider a cognitive radio network with multiple channels. The PU packets are supposed to have pre-emptive priority over the SU packets. In order to best satisfy the response requirement of the PUs, no buffer is deployed for the PU packets. On the other hand, in order to reduce the blocking rate of the SUs, a buffer is prepared for the SU packets. There is a central controller which can allocate the channels for the PUs and the SUs in the network [8].

Normally, when there is no packet to be transmitted, all of the channels will be not activated (also say "be closed") for network resource conservation. When there is a PU packet to be transmitted, due to the priority of the PU packets, the central controller will activate all of the channels, and these active channels will be aggregated into one bonding channel for the transmission of this PU packet.

However, when there are only SU packets in the system to be transmitted, the central controller will activate and aggregate a part of channels for the transmission of the SU packets. The motivation for this kind of channel bonding strategy can be presented as follows: Firstly, the remainder inactivated (also say "closed" in the following) channels can be reserved for the network management and maintenance; Secondly, the network resource and communication energy can be conserved effectively; Thirdly, the interference to the PUs can be reduced with less activated channels for the SUs.

In the proposed channel bonding strategy, the number of active channels for the SUs is supposed to be dependent on the number of SU packets in the system. We assume the buffer capacity of the SUs is $K (K > 0)$, so the system can hold at most $(K + 1)$ SU packets. Under the condition that there is no PU packet in the system, if the number of SU packets in the system is $H$, the occupy ratio for the SU packets in the system is $H / (K + 1)$. We denote the number of channels in the system as $N_c$. Since the number of active channels can only be integer, we apply a ceiling function $\lceil HN_c / (K + 1) \rceil$ to calculate the number of channels to be activated and aggregated into one bonding channel.
Additionally, considering the priority of the PU packets, if a PU packet arrives at the system during the transmission period of an SU packet, the transmission of the SU packet will be interrupted by the newly arriving PU packet, and the central controller will activate all of the channels for the transmission of this PU packet.

Moreover, we assume the interrupted SU packets have higher priorities than both the SU packets already in the buffer and the newly arriving SU packets. It means that the interrupted SU packet will return back to the head of the buffer. If the buffer of the SUs is full, the SU packet queueing at the end in the buffer will be forced to leave the system to make room for the interrupted SU packet. Given that there is only one vacancy in the buffer, if the arrival of an SU packet and the interruption of an SU packet occur simultaneously, the interrupted SU packet will join the system, and the newly arriving SU packet will be blocked by the system.

From the discussions above, we conclude that the PU packet will be transmitted by the bonding channel composed of all the channels. So, for the PU packets, the number of aggregated channels will be fixed. The SU packets will be transmitted by the bonding channel composed of channels in part, and the number of aggregated channels is dependent on the number of SU packets in the system. So, for the SU packets, the number of aggregated channels is adjustable. We call this channel bonding strategy as an adjustable channel bonding strategy in this paper.

### 2.2. System Model

We assume a slotted timing structure in which the time axis is segmented into a sequence of equal intervals, called slots. Suppose the time axis is ordered by \( t = 1, 2, \ldots \). To clarify the arrival and departure process of the packets, taking \( t = n \) as an example, we suppose the packets arrive immediately after the beginning instant of a slot (during the interval \((n,n^+)\)), and depart just prior to the end of a slot (during the interval \((n^-, n)\)).

We assume the arrival intervals of the PU packets and the SU packets follow geometrical distributions with parameters \( \lambda_1 (\bar{\lambda}_1 = 1 - \lambda_1, 0 < \lambda_1 < 1) \) and \( \lambda_2 (\bar{\lambda}_2 = 1 - \lambda_2, 0 < \lambda_2 < 1) \), respectively. Moreover, we assume the transmission time (in slots) of a PU packet in the bonding channel aggregated with all the \( N_c \) channels follows a geometrical distribution with parameter \( \mu_1 (\bar{\mu}_1 = 1 - \mu_1, 0 < \mu_1 < 1) \), and the transmission time (in slots) of an SU packet in each channel follows a geometrical distribution with parameter \( \mu_2 \). We call \( \mu_1 \) and \( \mu_2 \) the transmission rates (packets/slot) in this paper.

According to the adjustable channel bonding strategy proposed in this paper, we suppose that the actual transmission rate of an SU packet is dependent on the number \( H \) of SU packets in the system. The actual transmission rate \( \xi_{(H)} (\bar{\xi}_{(H)} = 1 - \xi_{(H)}) \) (packets/slot) can be given as follows:

\[
\xi_{(H)} = \left[ H N_c / (K + 1) \right] \mu_2.
\]  

(1)

In order to guarantee \( 0 < \xi_{(H)} < 1 \), we assume \( 0 < N_c \mu_2 < 1 \). Moreover, we define the traffic intensity of the PU packets as \( \rho_1 = \lambda_1 / \mu_1 \) (\( \rho_1 > 0 \)).

We denote

- \( L_n \) = the number of total packets in the system at the instant \( t = n^+ \).
- \( L_n^{(1)} \) = the number of PU packets in the system at the instant \( t = n^+ \).

Then, \( \{ L_n, L_n^{(1)} \} \) constitutes a two-dimensional discrete-time Markov chain with the
state space \( \Omega \) given as follows:

\[
\Omega = (0,0) \cup \{(i, j) : 1 \leq i \leq K + 1, \ j = 0, 1\},
\]

where \((0,0)\) denotes the state that there is no packet in the system; \((i, 0)\) denotes the state that there are \(i\) packets in the system, all these are SU packets; \((i, 1)\) denotes the state that there are \(i\) packets in the system, including one PU packet and \((i - 1)\) SU packets.

3. Performance Analysis

3.1. Transition Probability Matrix

Let \( P \) be the state transition probability matrix of the two-dimensional discrete-time Markov chain. We define the system level as the total number of packets, including PU packets and SU packets, in the system. According to the state transition of the Markov chain, \( P \) can be given in a \((K + 2) \times (K + 2)\) block-structure form as follows:

\[
P = \begin{pmatrix}
P_{0,0} & P_{0,1} & P_{0,2} & \cdots & 0 \\
P_{1,0} & P_{1,1} & P_{1,2} & \cdots & 0 \\
P_{2,0} & P_{2,1} & P_{2,2} & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
0 & P_{K,K-1} & P_{K,K} & \cdots & P_{K,K+1} \\
P_{K+1,K} & P_{K+1,K+1} & \cdots & \cdots & \cdots 
\end{pmatrix},
\]

where \(P_{u,v}\) is the transition probability sub-matrix from the system level \(u\) to \(v\), \((u = 0, 1, \ldots, K + 1, \ v = 0, 1, \ldots, K + 1)\). \(P\) can be discussed according to different system levels as follows.

(i) For the system level \(u = 0\), i.e., there is no packet in the system at the instant \(t = n^+\), the number of packets in the system at the instant \(t = (n + 1)^+\) can be \(v = 0, 1, 2\). So, for the system level \(u = 0\), there are three non-zero sub-blocks in \(P\) as follows:

When \(v = 0\), it means there is no packet arrival during the interval \((n + 1), (n + 1)^+)\), so \(P_{0,0}\) is a special vector with only one scalar value given by

\[
P_{0,0} = \lambda_1 \lambda_2.
\]

When \(v = 1\), it means there is one packet arrival (PU packet or SU packet) during the interval \((n + 1), (n + 1)^+)\), so \(P_{0,1}\) is a row vector with two elements given by

\[
P_{0,1} = (\lambda_1 \lambda_2, \lambda_1 \lambda_2).
\]

When \(v = 2\), it means there are two packet arrivals (one PU packet and one SU packet) during the interval \((n + 1), (n + 1)^+)\), so \(P_{0,2}\) is a row vector with two elements given by

\[
P_{0,2} = (0, \lambda_1 \lambda_2).
\]

(ii) For the system level \(u = 1\), i.e., there is only one packet in the system at the instant \(t = n^+\), the number of packets in the system at the instant \(t = (n + 1)^+\) can be \(v = 0, 1, 2, 3\). So, for the system level \(u = 1\), there are four non-zero sub-blocks in \(P\) as follows:

When \(v = 0\), it means the number of packets in the system is reduced by one. That is
to say, the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves, at the same time, there is no packet arrival during the interval \( ((n+1), (n+1)^+) \). So \( \mathbf{P}_{1,0} \) is a column vector with two elements given by

\[
\mathbf{P}_{1,0} = (\overline{\lambda}_1 \overline{\lambda}_2 \overline{\xi}_{(1)}, \overline{\lambda}_1 \overline{\lambda}_3 \mu_1)^T, 
\]

(6)

where \( T \) describes the transpose operator of the matrix.

When \( v = 1 \), it means that the number of packets in the system remains unchanged. The possibilities are described as follows: (I) Given that the PU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is no packet arrival or one PU packet arrival during the interval \( ((n+1), (n+1)^+) \). (II) Given that the SU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is no packet arrival during the interval \( ((n+1), (n+1)^+) \). (III) Given that the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves, there is one packet arrival (PU packet or SU packet) during the interval \( ((n+1), (n+1)^+) \). So \( \mathbf{P}_{1,1} \) is a \( 2 \times 2 \) matrix given by

\[
\mathbf{P}_{1,1} = \begin{pmatrix}
\overline{\lambda}_1 (\overline{\lambda}_2 \overline{\xi}_{(1)} + \lambda_2 \overline{\xi}_{(1)}) & \lambda_1 \overline{\lambda}_2 \overline{\xi}_{(1)} \\
\overline{\lambda}_1 \lambda_2 \mu_1 & \overline{\lambda}_2 (\lambda_1 \mu_1 + \mu_1)
\end{pmatrix}. 
\]

(7)

When \( v = 2 \), it means that the number of packets in the system is increased by one. The possibilities are described as follows: (I) Given that the PU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is one SU packet arrival or two packet arrivals (one PU packet and one SU packet) during the interval \( ((n+1), (n+1)^+) \). (II) Given that the SU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is one packet arrival (PU packet or SU packet) during the interval \( ((n+1), (n+1)^+) \). (III) Given that the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves, there are two packet arrivals (one PU packet and one SU packet) during the interval \( ((n+1), (n+1)^+) \). So \( \mathbf{P}_{1,2} \) is a \( 2 \times 2 \) matrix given by

\[
\mathbf{P}_{1,2} = \begin{pmatrix}
\overline{\lambda}_1 \overline{\lambda}_2 \overline{\xi}_{(1)} & \lambda_1 (\overline{\lambda}_2 \overline{\xi}_{(1)} + \lambda_2 \overline{\xi}_{(1)}) \\
0 & \lambda_2 (\lambda_1 \mu_1 + \mu_1)
\end{pmatrix}. 
\]

(8)

When \( v = 3 \), it means that the number of packets in the system is increased by two. The only possibility is described as follows: The SU packet in the bonding channel at the instant \( t = n^+ \) does not leave, and there are two packet arrivals (one PU packet and one SU packet) during the interval \( ((n+1), (n+1)^+) \). So \( \mathbf{P}_{1,3} \) is a \( 2 \times 2 \) matrix given by

\[
\mathbf{P}_{1,3} = \begin{pmatrix}
0 & \lambda_1 \lambda_2 \overline{\xi}_{(1)} \\
0 & 0
\end{pmatrix}. 
\]

(9)

(iii) For the system level \( 2 \leq u \leq K - 1 \), i.e., there are \( u \) packets in the system at the instant \( t = n^+ \), the possible number of packets in the system at the instant \( t = (n+1)^+ \) is \( v = u - 1, u, u + 1, \) or \( u + 2 \). So, for the system level \( 2 \leq u \leq K - 1 \), there are four non-zero sub-blocks in \( \mathbf{P} \) as follows:

When \( v = u - 1 \), it means that the number of packets in the system is reduced by one. That is to say, the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves. At the same time, there is no packet arrival during the interval
((n+1), (n+1)\textsuperscript{+}). So, \( P_{u, u-1} \) is a \( 2 \times 2 \) matrix given by

\[
P_{u, u-1} = \begin{pmatrix} \lambda_1 \lambda_2 & 0 \\ \lambda_1 \lambda_2 \mu_1 & 0 \end{pmatrix}.
\] (10)

Similar to the matrix structures shown in Equations (7-9), the transition probability matrix \( P_{u, v} \) for \( v (v = u, u+1, u+2) \) can be given as follows:

When \( v = u \), \( P_{u, u} \) is a \( 2 \times 2 \) matrix given by

\[
P_{u, u} = \begin{pmatrix} \lambda_1 (\lambda_2 \bar{S} _{(u)} + \lambda_2 \bar{Z}(u)) & \lambda_1 \lambda_2 \bar{Z}(u) \\ \lambda_1 \lambda_2 \mu_1 & \lambda_2 (\lambda_1 \mu_1 + \bar{\mu}) \end{pmatrix}.
\] (11)

When \( v = u+1 \), \( P_{u, u+1} \) is a \( 2 \times 2 \) matrix given by

\[
P_{u, u+1} = \begin{pmatrix} \lambda_1 \lambda_2 \bar{Z} _{(u)} & \lambda_1 (\lambda_2 \bar{S} _{(u)} + \lambda_2 \bar{Z}(u)) \\ 0 & \lambda_2 (\lambda_1 \mu_1 + \bar{\mu}) \end{pmatrix}.
\] (12)

When \( v = u+2 \), \( P_{u, u+2} \) is a \( 2 \times 2 \) matrix given by

\[
P_{u, u+2} = \begin{pmatrix} 0 & \lambda_1 \lambda_2 \bar{Z} _{(u)} \\ 0 & 0 \end{pmatrix}.
\] (13)

(iv) For the system level \( u = K \), i.e., there are \( K \) packets in the system at the instant \( t = n^+ \). At the same time, there is only one vacancy in the buffer of the SUs. The number of packets in the system at the instant \( t = (n+1)^+ \) can be \( v = K-1, K \), or \( K+1 \). So, for the system level \( u = K \), there are three non-zero sub-blocks in \( P \) as follows:

When \( v = K-1 \), similar to the matrix structure shown in Equation (10), the transition probability matrix \( P_{K, K-1} \) is a \( 2 \times 2 \) matrix which can be given by

\[
P_{K, K-1} = \begin{pmatrix} \lambda_1 \lambda_2 \bar{Z} _{(K)} & 0 \\ \lambda_1 \lambda_2 \mu_1 & 0 \end{pmatrix}.
\]

When \( v = K \), similar to the matrix structure shown in Equation (11), the transition probability matrix \( P_{K, K} \) is a \( 2 \times 2 \) matrix which can be given by

\[
P_{K, K} = \begin{pmatrix} \lambda_1 (\lambda_2 \bar{S} _{(K)} + \lambda_2 \bar{Z}(K)) & \lambda_1 \lambda_2 \bar{Z}(K) \\ \lambda_1 \lambda_2 \mu_1 & \lambda_2 (\lambda_1 \mu_1 + \bar{\mu}) \end{pmatrix}.
\]

When \( v = K+1 \), it means that the number of packets in the system is increased by one. The possibilities are described as follows: (I) Given that the PU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is one SU packet arrival or two packet arrivals (one PU packet and one SU packet) during the interval \(((n+1), (n+1)^+)\). (II) Given that the SU packet in the bonding channel at the instant \( t = n^+ \) does not leave, there is one packet arrival (PU packet or SU packet) or two packets arrivals (one PU packet and one SU packet) during the interval \(((n+1), (n+1)^+)\). (III) Given that the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves, there are two packet arrivals (one PU packet and one SU packet) during the interval \(((n+1), (n+1)^+)\). So, \( P_{K, K+1} \) is a \( 2 \times 2 \) matrix given by

...
\[
\mathbf{P}_{K,K+1} = \begin{pmatrix}
\lambda_2 \xi(K) & \lambda_1 (1 - \lambda_2 \xi(K)) \\
0 & \lambda_2 (\lambda_1 \mu_1 + \overline{\mu}_1)
\end{pmatrix}.
\]

(v) For the system level \( u = K + 1 \), i.e., there are \((K+1)\) packets in the system at the instant \( t = n^+ \), and the buffer of the SUs is full, the number of packets in the system at the instant \( t = (n+1)^+ \) can be \( v = K \), or \( K + 1 \). So, for the system level \( u = K + 1 \), there are two non-zero sub-blocks in \( \mathbf{P} \) as follows:

When \( v = K \), similar to the matrix structure shown in Equation (10), the transition probability matrix \( \mathbf{P}_{K+1,K} \) is a \( 2 \times 2 \) matrix which can be given by

\[
\mathbf{P}_{K+1,K} = \begin{pmatrix}
\lambda_2 \xi(K+1) & 0 \\
0 & \lambda_2 \mu_1 + \overline{\mu}_1
\end{pmatrix}.
\]

When \( v = K + 1 \), it means that the number of packets in the system remains unchanged. The possibilities are described as follows: (I) Given that the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) does not leave, no matter how many packets arrive at the system during the interval \((n + 1) \), \((n + 1)^+\)\), the number of packets in the system remains unchanged. (II) Given that the packet (PU packet or SU packet) in the bonding channel at the instant \( t = n^+ \) leaves, there is one packet arrival (PU packet or SU packet) or two packet arrivals (one PU packet and one SU packet) during the interval \((n + 1) \), \((n + 1)^+\)\). So, \( \mathbf{P}_{K+1,K+1} \) is a \( 2 \times 2 \) matrix given by

\[
\mathbf{P}_{K+1,K+1} = \begin{pmatrix}
\lambda_1 (\lambda_2 \xi(K+1) + \xi(K+1)) & \lambda_1 \\
\lambda_1 \lambda_2 \mu_1 & \lambda_1 \mu_1 + \overline{\mu}_1
\end{pmatrix}.
\]

Up to now, all the elements in the transition probability matrix \( \mathbf{P} \) have been given.

### 3.2. Steady-State Distribution

The structure of the transition probability matrix \( \mathbf{P} \) indicates that the two-dimensional Markov chain \( \{ L_n, I_n^{(1)} \} \) is non-periodic, irreducible and positive recurrent. The steady-state distribution \( \pi_{i,j} \) of the two-dimensional Markov chain is defined as follows:

\[
\pi_{i,j} = \lim_{n \to \infty} P \{ L_n = i, I_n^{(1)} = j \}.
\] (14)

Let \( \Pi_i \) be the steady-state probability vector for the system being at level \( i \). \( \Pi_i \) can be given as follows:

\[
\Pi_i = \begin{cases}
\pi_{0,0} , & i = 0 \\
(\pi_{i,0}, \pi_{i,1}) , & 1 \leq i \leq K + 1.
\end{cases}
\] (15)

Moreover, according to the balance equation with the normalization condition [10], we have

\[
\left\{ \begin{array}{l}
(\Pi_0, \Pi_1, \ldots, \Pi_K, \Pi_{K+1}) \mathbf{P} = (\Pi_0, \Pi_1, \ldots, \Pi_K, \Pi_{K+1}) \\
(\Pi_0, \Pi_1, \ldots, \Pi_K, \Pi_{K+1}) \mathbf{e} = \mathbf{1},
\end{array} \right.
\] (16)

where \( \mathbf{e} \) is a one’s column vector with appropriate dimension.

By substituting Equation (15) to Equation (16) and applying the Gaussian-Seidel method, we can obtain the steady-state distribution \( \pi_{i,j} \) defined in Equation (14).
4. Performance Measures and Numerical Experiments

4.1. Performance Measures

We define the blocking rate $\beta$ of the SUs as the number of newly arriving SU packets that are blocked by the system per slot. A newly arriving SU packet will be blocked by the system when the bonding channel is being occupied, while the buffer of the SUs is full. Therefore, the blocking rate $\beta$ of the SUs can be given as follows:

$$\beta = \lambda_2 (\bar{\xi}_{(K+1)} + \xi_{(K+1)} \lambda_1) \pi_{K+1,0} + \lambda_2 (\bar{\mu}_1 + \mu_1 \lambda_1) \pi_{K+1,1} + \lambda_1 \lambda_2 \bar{\xi}_{(K)} \pi_{K,0}. \tag{17}$$

We define the normalized throughput $\theta$ of the SUs as the number of SU packets transmitted successfully per slot. An SU packet can be transmitted successfully if and only if this SU packet is not blocked by the system when it arrives at the system, and not forced to leave the system before the transmission is successfully completed. Therefore, the normalized throughput $\theta$ of the SUs can be given by

$$\theta = \lambda_2 - \beta - \lambda_1 \bar{\xi}_{(K+1)} \pi_{K+1,0}. \tag{18}$$

We define the latency of an SU packet as the time period from the instant an SU packet arrives at the system to the instant that the SU packet is successfully transmitted.

Let $L_n^{(2)}$ be the number of SU packets in the system at the instant $t = n^+$, and let $L^{(2)} = \lim_{n \to \infty} L_n^{(2)}$ be the number of SU packets in the steady state. We can get the average number $E[L^{(2)}]$ of $L^{(2)}$ as follows:

$$E[L^{(2)}] = \sum_{i=0}^{K+1} i P\{L^{(2)} = i\} = \sum_{i=0}^{K} i (\pi_{i+1,1} + \pi_{i,0}) + (K + 1) \pi_{K+1,0}. \tag{19}$$

By using Little’s law, the average latency $\delta$ of the SUs can be given as follows:

$$\delta = \frac{E[L^{(2)}]}{\lambda_2 - \beta - \lambda_1 \bar{\xi}_{(K+1)} \pi_{K+1,0}}. \tag{20}$$

We define the closed channel ratio $\tau$ as the probability that one channel is closed. The closed channel ratio $\tau$ is one of the important performance measures to evaluate the efficiency of the proposed channel bonding strategy. The larger the closed channel ratio is, the more the network resource and communication energy will be conserved, and the smaller the interference to the PUs will be. We note that this measurement is related to the proportion of the closed channels. The closed channel ratio $\tau$ can be given as follows:

$$\tau = \sum_{i=0}^{K+1} \pi_{i,0} \frac{N_c - \lceil iN_c/(K+1) \rceil}{N_c}, \tag{21}$$

where $N_c$ is the number of channels in the system, $K$ is the buffer capacity of the SUs, and $\pi_{i,0}$ is the probability that the spectrum is not occupied by the PUs.

4.2. Numerical Experiments

In this section, we firstly describe the change trend of the actual transmission rate of the SU packets, then we show the influence of the buffer capacity of the SUs on the system performance, such as the blocking rate, the normalized throughput, the average latency of the SUs and the closed channel ratio.
In the numerical experiments, unless otherwise specified, the parameters are set as follows: We assume the transmission rate $\mu_1$ and the traffic intensity $\rho_1$ of the PU packets are $\mu_1 = 0.1$ and $\rho_1 = 0.2, 0.8$ to analyze the impact of the PUs on the SUs. We set the arrival rate $\lambda_2$ of the SU packets as $\lambda_2 = 0.2, 0.3$ to investigate the influence of the arrival of the SUs on the system performance. Moreover, we suppose the number $N_c$ of channels in the system is $N_c = 15, 30$ to investigate how the number of channels influences the system performance. Additionally, with the constraint of $0 < N_c \mu_2 < 1$, we set the transmission rate $\mu_2$ of the SU packets on single channel as $\mu_2 = 0.03$.

Figure 1 depicts how the actual transmission rate $\xi_{(H)}$ of the SU packets changes versus the number $H$ of packets in the system for different numbers $N_c$ of channels. From Figure 1, we find that for the same number $H$ of packets in the system, as the number $N_c$ of channels increases, the actual transmission rate $\xi_{(H)}$ of the SU packets will show an increasing tendency. On the other hand, for the same number $N_c$ of channels, as the number $H$ of packets in the system increases, the actual transmission rate $\xi_{(H)}$ of the SU packets will also show an upward tendency. Both of the two trends mentioned above are self-evident. Specially, we find that the change trend of the transmission rate $\xi_{(H)}$ of the SU packets is not smooth, this is because we introduced the operation $\left\lceil \cdot \right\rceil$ when calculating the actual transmission rate $\xi_{(H)}$ of the SUs.

Figures 2-4 illustrate how the blocking rate $\beta$, the normalized throughput $\theta$ and the average latency $\delta$ of the SUs change with respect to the buffer capacity $K$ of the SUs for different parameter settings. From Figures 2-4, when other parameters are given, we see that as the buffer capacity $K$ of the SUs increases, the blocking rate $\beta$ of the SUs shows a declining tendency, while the normalized throughput $\theta$ and the average latency $\delta$ of the SUs show
increasing tendencies. This is because the larger the buffer capacity of the SUs is, the more likely it is that a newly arriving SU packet can join the system, so the lower the blocking rate and the greater the normalized throughput of the SUs will be. Moreover, as the buffer capacity of the SUs increases, more SU packets can stay in the system, which will induce a longer average latency of the SUs.

Figure 2. Blocking rate $\beta$ of the SUs vs. buffer capacity $K$ of the SUs.

Figure 3. Normalized throughput $\theta$ of the SUs vs. buffer capacity $K$ of the SUs.
increasing tendency. The intuitive reason is that as the number of channels increases, the actual transmission rate of the SU packets will increase. Then more SU packets will be transmitted successfully. So, the normalized throughput of the SUs will increase. At the same time, the number of SU packets in the system will decrease, which will lower both the blocking rate and the average latency of SUs.

Figure 5 demonstrates how the closed channel ratio $\tau$ changes with respect to the buffer capacity $K$ of the SUs for different parameter settings.

As illustrated in Figure 5, when other parameters are given, we see that as the buffer capacity $K$ of the SUs increases, the closed channel ratio $\tau$ shows a decreasing tendency. The reason is that the larger the buffer capacity of the SUs is, the greater the number of SU packets that can join the system is, the less likely it is that one channel is closed, and the lower the closed channel ratio will be.

On the other hand, Figure 5 also shows that, when other parameters are fixed, as the traffic intensity $\rho_1$ of the PU packets increases, the closed channel ratio $\tau$ shows a declining tendency. This is because the higher the traffic intensity of the PU packets is, the more likely it is that all the channels are occupied by a PU packet, so the probability for one channel being closed will be less. This will lead to a decrease in the closed channel ratio.

Moreover, when other parameters are given, as the arrival rate $\lambda_1$ of the SU packets increases, the closed channel ratio $\tau$ shows a downward tendency. This is because that the greater the arrival rate of the SU packets is, the more the SU packets will join the system, and the more the channels will be activated, then the closed channel ratio will decrease.

Additionally, when other parameters are given, as the number $c$ of channels increases, the closed channel ratio $\tau$ also shows an increasing tendency. The reason may be that, as the number of channels increases, it is more quickly for an SU packet to be

On the other hand, when other parameters are fixed, we observe that as the traffic intensity $\rho_1$ of the PU packets increases, the blocking rate $\beta$ and the average latency $\delta$ of the SUs show rising trends, while the normalized throughput $\theta$ of the SUs shows a descending trend. This is because the heavier the traffic intensity of the PU packets is, the more likely it is that all the channels are occupied by a PU packet. As a result, more possible is that the buffer is overflow, then more SU packets will be blocked by the system, i.e., less SU packets will be transmitted successfully. So, the blocking rate of the SUs will be greater and the normalized throughput of the SUs will be smaller. Additionally, more SU packets in the system will inevitably result in a longer average latency of the SUs.

Moreover, when other parameters are given, we find that as the arrival rate $\lambda_2$ of the SU packets increases, the blocking rate $\beta$ and the normalized throughput $\theta$ of the SUs show upward tendencies, the reason is that the greater the arrival rate of the SU packets is, the more the SU packets will join the system, and the number of SU packets being blocked by the system will increase, at the same time, the number of SU packets being transmitted successfully will also increase. So both the blocking rate and the normalized throughput of the SUs will increase. It is worth noting that when other parameters are given, the average latency $\delta$ of the SUs will decrease as the arrival rate $\lambda_2$ of the SU packets increases from 0.2 to 0.3. The reason for this interesting phenomena may be that when the arrival rate of the SU packets is greater, more SU packets will join the system, and more channels will be activated to transmit these SU packets, then the average transmission rate for single SU packet will increase, obviously, this will result in a decrease in the average latency of the SUs.

Additionally, when other parameters are fixed, we conclude that as the number of channels increases, the blocking rate $\beta$ and the average latency $\delta$ of the SUs show descending tendencies, while the normalized throughput $\theta$ of the SUs shows an
increasing tendency. The intuitive reason is that as the number of channels increases, the actual transmission rate of the SU packets will increase. Then more SU packets will be transmitted successfully. So, the normalized throughput of the SUs will increase. At the same time, the number of SU packets in the system will decrease, which will lower both the blocking rate and the average latency of SUs.

Figure 5 demonstrates how the closed channel ratio $\tau$ changes with respect to the buffer capacity $K$ of the SUs for different parameter settings.

![Figure 5. Closed channel ratio $\tau$ vs. buffer capacity $K$ of the SUs.](image)

As illustrated in Figure 5, when other parameters are given, we see that as the buffer capacity $K$ of the SUs increases, the closed channel ratio $\tau$ shows a decreasing tendency. The reason is that the larger the buffer capacity of the SUs is, the greater the number of SU packets that can join the system is, the less likely it is that one channel is closed, and the lower the closed channel ratio will be.

On the other hand, Figure 5 also shows that, when other parameters are fixed, as the traffic intensity $\rho_1$ of the PU packets increases, the closed channel ratio $\tau$ shows a declining tendency. This is because the higher the traffic intensity of the PU packets is, the more likely it is that all the channels are occupied by a PU packet, so the probability for one channel being closed will be less. This will lead to a decrease in the closed channel ratio.

Moreover, when other parameters are given, as the arrival rate $\lambda_2$ of the SU packets increases, the closed channel ratio $\tau$ shows a downward tendency. This is because that the greater the arrival rate of the SU packets is, the more the SU packets will join the system, and the more the channels will be activated, then the closed channel ratio will decrease.

Additionally, when other parameters are given, as the number $N_c$ of channels increases, the closed channel ratio $\tau$ also shows an increasing tendency. The reason may be that, as the number of channels increases, it is more quickly for an SU packet to be
transmitted. Then the number of SU packets in the system will decrease, and the number of channels being closed will increase. This will make the closed channel ratio greater.

5. Nash Equilibrium and Social Optimization

In this section, we firstly discuss the SU packets’ behavior in Nash equilibrium, and then we turn our attention to analyze the social optimization for the adjustable channel bonding strategy proposed in this paper.

Some assumptions preparing for the optimizations are given as follows:

(i) When an SU packet arrives at the system, it can decide whether or not to join the system. But it is not possible for this SU packet being able to observe the number of packets in the system before making a decision. Moreover, the decision to join is irrevocable, and reneging is not permitted.

(ii) An SU packet’s benefit from a successful transmission is $R$.

(iii) When an SU packet chooses to join the system, it may be blocked or dropped, so the transmission of this SU packet is not guaranteed. As a result, we refer to an SU packet joining the system as a trial, and introduce a cost $T(T < R)$ associated with each trial of an SU packet. That is to say, when an SU packet decides to join the system, it will firstly pay a cost $T$. We note that $T$ is not a transfer payment, but a real cost.

(iv) The potential arrival rate of the SU packets is denoted as $\Lambda$.

From the assumptions above, we conclude that the key issue for our model is that whether or not an SU packet tries to join the system. We therefore consider an optimal strategy characterized by the probability $q$ ($0 \leq q \leq 1$) that an SU packet tries to join the system.

5.1. Nash Equilibrium Strategy

From the analysis in Sections 3 and 4, we can obtain the expression for the probability $\epsilon(\lambda_2)$ that an SU packet can be successfully transmitted as follows:

$$
\epsilon(\lambda_2) = 1 - \frac{\beta + \lambda_2}{\lambda_2},
$$

where $\beta$ is the blocking rate of the SUs given in Equation (17), and $\lambda_2$ ($0 < \lambda_2 < 1$) is the arrival rate of the SU packets.

We further define individual net benefit function $W_I(\lambda_2)$ for one SU packet who tries to join the system as follows:

$$
W_I(\lambda_2) = \epsilon(\lambda_2)(R - T) - (1 - \epsilon(\lambda_2))T = \epsilon(\lambda_2)R - T.
$$

Motivated by the concept of Nash equilibrium [4], we denote the equilibrium trial probability by $q_e$, and then the equilibrium trial rate is $\lambda_e = q_e\Lambda$. With $q_e$ and $\lambda_e$, the Nash equilibrium will be reached. In other words, no individual SU packet has any incentive to deviate unilaterally from the equilibrium trial rate.

As the expression of $\epsilon(\lambda_2)$ in Equation (23) is complex, we investigate the monotonicity of $W_I(\lambda_2)$ with numerical experiments. With the same parameters used in Figures 2-5, by setting $N_c = 15$, $\Lambda = 0.5$, $R = 3$ and $T = 2$ as an example, we show how the individual net benefit $W_I(\lambda_2)$ changes with respect to the arrival rate $\lambda_2$ of the SU packets in Figure 6.
From Figure 6, we find that the individual net benefit monotonically decreases as the arrival rate of the SU packets increases. Then we discuss the Nash equilibrium strategy for two cases (to avoid a trivial solution, we assume \( \lim_{\lambda_2 \to 0} W_i(\lambda_2) > 0 \)).

For \( W_i(\Lambda) \geq 0 \), even if all the SU packets join the system, the net benefit will be non-negative. That is to say, regardless of how other packets do, one SU packet's best action is trying to join the system. In this case, what trying to join the system with the trial probability \( q_e = 1 \) is an equilibrium strategy, and the corresponding equilibrium trial rate is \( \lambda_e = \Lambda \).

For \( W_i(\Lambda) < 0 \), when all the SU packets join the system, the net benefit will be negative. In this case, if \( q_e = 1 \), then an SU packet who tries to join the system will get a negative net benefit. On the other hand, if \( q_e = 0 \), an SU packet who tries to join the system will get a positive benefit, which is a better outcome than not trying to join. Obviously, neither of the two cases mentioned above is an equilibrium strategy. Therefore, there exists a unique equilibrium strategy \( q_e = \lambda_e / \Lambda \), where the equilibrium trial rate \( \lambda_e \) is an equilibrium point obtained by solving \( W_i(\lambda_e) = 0 \).

In the following, based on the results shown in Figure 6, we tabulate the trial rate and trial probability with Nash equilibrium strategy in Table 1.

In Figure 6, we note that the accuracy of the arrival rate is 0.01, and the equilibrium point is difficult to be given exactly. Considering the monotonically decreasing property of the individual net benefit function, we present the value ranges of the trial rate and trial probability with Nash equilibrium strategy in Table 1. Taking \( \rho_1 = 0.2 \) and \( K = 2 \) in Figure 6 as an example, we find that the solution \( \lambda_e \) for \( W_i(\lambda_e) = 0 \) is in \([0.39, 0.40]\). So, in Table 1, we denote the lower boundary for the equilibrium trial rate as \( \text{Min} = 0.39 \), and the upper boundary for the equilibrium trial rate as \( \text{Max} = 0.40 \).
In order to oblige the SU packets to adopt the socially optimal strategy, we can apply a pricing mechanism to reduce the equilibrium trial probability and the corresponding equilibrium trial rate.

5.3. Pricing Mechanism

We assume the central controller acts as a pricing agent and will impose an admission fee on all the trials of the SU packets in a centralized cognitive radio network with the adjustable channel bonding strategy.

When an admission fee \( f \) is imposed, the benefit \( P_W(\lambda_2) \) of an SU packet trial will be given as follows:

\[
P_W(\lambda_2) = (22) \left( R - T \right) f \cdot \left( 1 - \frac{\lambda_2}{\lambda} \right) - \frac{\lambda_2}{\lambda} - \frac{f \cdot \lambda_2}{2}.
\]

We note that the social objective is to maximize the total benefit of the central controller and the SUs. By considering the admission fee \( f \), we redefine the social benefit function \( SW(\lambda_2) \) as follows:

\[
SW(\lambda_2) = \left( R - T \right) f \cdot \left( 1 - \frac{\lambda_2}{\lambda} \right) - \frac{\lambda_2}{\lambda} - \frac{f \cdot \lambda_2}{2}.
\]

By comparing Equations (28) and (24), we find that the social benefit function when an admission fee is imposed is the same as the social benefit function without admission fee. It means that the admission fee just implies a transfer of benefit from the SU packets to the central controller. It comes to the conclusion that imposing an admission fee does not influence the socially optimal strategy.

With the \( \lambda^* \) obtained in Equation (25), by setting \( \lambda_2^* = \lambda \) in Equation (27), we can calculate the admission fee \( f \) with \( P_W(\lambda^*) = 0 \).

### Table 1. Numerical results with Nash equilibrium strategy.

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Trial rate ( \lambda_e )</th>
<th>Trial probability ( q_e )</th>
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</thead>
<tbody>
<tr>
<td>Traffic intensity ( \rho_1 )</td>
<td>Buffer capacity ( K )</td>
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</table>

### Table 2. Numerical results with socially optimal strategy.

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Trial rate ( \lambda^* )</th>
<th>Trial probability ( q^* )</th>
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<tbody>
<tr>
<td>Traffic intensity ( \rho_1 )</td>
<td>Buffer capacity ( K )</td>
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<td>4</td>
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<td>6</td>
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<td>0.8</td>
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<td>8</td>
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</tbody>
</table>
In order to oblige the SU packets to adopt the socially optimal strategy, we can apply a pricing mechanism to reduce the equilibrium trial probability and the corresponding equilibrium trial rate.

5.3. Pricing Mechanism

We assume the central controller acts as a pricing agent and will impose an admission fee on all the trials of the SU packets in a centralized cognitive radio network with the adjustable channel bonding strategy.

When an admission fee $f$ is imposed, the benefit $W_p(\lambda_2)$ of an SU packet trial will be given as follows:

$$W_p(\lambda_2) = \varepsilon(\lambda_2)(R - T - f) - (1 - \varepsilon(\lambda_2))(T + f) = \varepsilon(\lambda_2)R - T - f.$$  (27)

We note that the social objective is to maximize the total benefit of the central controller and the SUs. By considering the admission fee $f$, we redefine the social benefit function $W_s(\lambda_2)$ as follows:

$$W_s(\lambda_2) = \lambda_2(\varepsilon(\lambda_2)R - T - f) + \lambda_2 f = \lambda_2(\varepsilon(\lambda_2)R - T).$$  (28)

By comparing Equations (28) and (24), we find that the social benefit function when an admission fee is imposed is the same as the social benefit function without admission fee. It means that the admission fee $f$ just implies a transfer of benefit from the SU packets to the central controller. It comes to the conclusion that imposing an admission fee $f$ does not influence the socially optimal strategy.

With the $\lambda^*$ obtained in Equation (25), by setting $\lambda_2 = \lambda^*$ in Equation (27), we can calculate the admission fee $f$ with $W_p(\lambda^*) = 0$. 

![Figure 7. Social net benefit $W_s(\lambda_2)$ vs. arrival rate $\lambda_2$ of the SU packets.](image)
In the case of $\lambda^* < \Lambda$, we can obtain the admission fee $f$ as follows:

$$f = \varepsilon(\lambda^*) R - T.$$  

(29)

In the case of $\lambda^* = \Lambda$, we can obtain the admission fee $f$ as follows:

$$f = \varepsilon(\Lambda) R - T.$$  

(30)

For example, with the socially trial rate $\lambda^*$ obtained in Table 2, we give the admission fees $f$ for different parameter settings in Table 3. In Table 3, the estimates of the admission fee $f$ are accurate to four decimal places.

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Socially trail rate $\lambda^*$</th>
<th>Admission fee $f$</th>
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<tr>
<td>Traffic intensity $\rho_1$</td>
<td>Buffer capacity $K$</td>
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### 6. Conclusions

In this paper, we proposed an adjustable channel bonding strategy for a centralized cognitive radio network. The main idea is that the central controller can aggregate a part of channels for serving the SUs according to the number of packets available in the system. We built a priority queue by considering an adjustable transmission rate. We constructed a two-dimensional Markov chain and gave the transition probability matrix to analyze the system performance. We derived the formulas for the blocking rate, the normalized throughput, the average latency of the SUs and the closed channel ratio, and then we gave numerical experiments to show the influence of the buffer capacity of the SUs on the system performance. Moreover, considering an SU packet’s join as a trial, we investigate the Nash equilibrium and the social optimization for the SUs. At last, in order to oblige the SU packets to adopt the socially optimal strategy, we proposed a pricing mechanism by imposing an admission fee on the SU packets in a centralized cognitive radio network with the adjustable channel bonding strategy.

### Acknowledgments

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An Adjustable Channel Bonding Strategy in Centralized Cognitive Radio Networks

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A Model to Analyze Airbase Oil Inventory System with Endogenous Lead Time

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Abstract: In this paper, we analyze airplane oil inventory problem of a base in the Republic of Korea Air Force (ROKAF) which should keep maintaining an appropriate inventory level of oil for the efficient operation of flights. In ROKAF, airplane oil supply is ordered (or delivered) at a constant time period, which means reorder point is not stochastic but deterministic. Since the airplane oil inspection time occupies large proportion of time between a reorder point and the time the oil is available for usage, it is imperative to analyze the inspection time which causes delay in fulfilling an order. In addition, if demands increase dramatically because of a war or an unexpected training, the reorder point could be shortened than in normal situations. In such abnormal circumstances, the shortened reorder point may require more delay for order fulfillment. In this setting, the inspection time endogenously determines the replenishment lead time (or supply lead time) and the pattern of the inventory. Meanwhile, since the procedure of airplane oil inspection is composed of several steps, it may be better that the distributions of inspection times are treated as phase-type distributions. In this context, we use D/PH/1/m queue and counting process of D-BMAP (Discrete-time Batch Markovian Arrival Process) with 2-class to analyze the delay (lead time) for order fulfillment. Using this model, we find a way to derive decision variables such as the number of inventory replenishments and average fill rate.

Keywords: Airbase oil inventory, D/PH/1/m, D-BMAP, endogenous lead time, phase-type distribution, queueing theory.

1. Introduction

In many cases, product cannot be available for immediate use until it is inspected to decide whether it is appropriate for usage or not. Likewise, the airplane oil inspection is very important for the airbase to prevent an accident like a drop or a breakout. Since the airplane oil inspection should be thorough, the airplane oil inspection in an airbase is composed of several phases. Moreover, the airplane oil inspection time determines the replenishment lead time (or supply lead time) and the pattern of the inventory. This inspection/replenishment lead time is affected by order quantity and inspection time, resulting in endogenous lead time which occurs by internal factors of system (e.g. production time delay). Most of the studies on the inventory systems have focused on exogenous lead time which occurs by exterior factors of system (e.g. delivery time delay, administration time delay). The model with exogenous lead time can be found in the literature [11, 14, 16-19] and references therein. However, very few results have been

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