Product Customization and Price Competition on the Internet

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The Internet provides an unprecedented capability for sellers to learn about their customers and offer custom products at special prices. In addition, customization is more feasible today because of advances in manufacturing technologies that have improved sellers’ manufacturing flexibility. We first develop a model of product customization and flexible pricing to incorporate the salient roles of the Internet and flexible manufacturing technologies in reducing the costs of designing and producing tailored consumer goods. We show how a monopoly seller may earn the highest profits by producing both standard and custom products and can raise prices for both types of products as customization and information collection technologies improve.

Simultaneous adoption of customization in a duopoly reduces the differentiation between their standard products but does not intensify price competition. Compared with a two-facility monopolist, the duopoly may underinvest in customization. Consumer surplus improves after sellers adopt customization but does not monotonically increase as customization technologies advance. When firms face a fixed entry cost and adopt customization sequentially, the first entrant always achieves an advantage and may be able to deter subsequent entry by choosing its customization scope strategically.

1. Introduction

New information technologies are transforming the dynamics of producing and marketing consumer goods. The Internet and consumer-tracking technologies such as online registration, cookies, and collaborative filtering allow sellers to better understand each individual customer’s tastes at very low costs. The ease of collecting information about consumer preferences makes large-scale custom product design a reality in many product categories. Meanwhile, advances in flexible manufacturing systems (FMS), computer-aided design/manufacturing (CAD/CAM), and just-in-time (JIT) have provided a new production choice that integrates the efficiency of assembly lines and the flexibility of numerically controlled machines in a job shop (Gerwin 1982, Seidmann 1993). This improvement in manufacturing flexibility allows mass customization of consumer products without significantly compromising cost efficiency. Not surprisingly, mass customization has begun to erode the domain of mass-produced standard items.

Flexible pricing has also become ubiquitous on the Internet because the considerable costs of implementing multiple prices (the so-called menu costs) in brick-and-mortar stores have essentially disappeared in today’s electronic markets (Business Week 1998, 2000). Reduced menu costs and customization enhance sellers’ ability to price discriminate; premium prices can be charged for custom goods because tailored product features better comply with buyers’ needs. Many firms are already customizing and price differentiating their products. Automobile makers such as Ford...
and Toyota offer friendly interfaces through which buyers can “design” their own cars. Computer vendors such as Dell and Compaq allow customers to configure their own machines online. Apparel vendor Gap.com takes custom orders besides offering clothes of standard sizes and colors.

There exists abundant empirical and qualitative literature regarding the various substantial impacts of the Internet on a firm’s marketing strategies (Hagel and Rayport 1997, Pine et al. 1995, Rayport and Sviokla 1994). Yet related analytical research is still rare and mostly focuses on the effects of reduced buyer search costs on market price (Bakos 1997, Lal and Sarvary 1999). Extending the classic model of Salop (1979), we incorporate the prominent features of information and flexible manufacturing technologies and show how they may allow firms to customize their products by reducing the related costs. Because the enabling technologies evolve constantly, we highlight the dynamic impacts of customization on firms’ product and pricing strategies, consumer welfare, and social welfare. The following issues are our focus:

1. What is a seller’s optimal mix of customized and standard products, and how do advances in related technologies affect its product mix and pricing?

2. In a duopoly, how does adoption of customization affect price competition? Will consumers, as a group, always benefit from technological progress?

3. In a sequential duopoly, how does the first mover achieve an advantage over the second adopter? Can excess customization capability be used to deter entry?

Our research draws upon existing literature in spatial product differentiation, flexible manufacturing, and entry deterrence. The study of spatial product differentiation dates back to Hotelling’s (1929) seminal paper and was generalized by Lancaster (1975) and Salop (1979). In all these models, each brand is represented by one point in the product space and directly competes with its immediately neighboring brands (“localized competition”). These models have the desired property of explicitly addressing product attributes but cannot directly capture the increasingly popular practices in electronic markets: customization and price discrimination.

A branch of the spatial theory called delivered pricing (Beckman 1976, Thisse and Vives 1988) treats customization as redesigning a basic product to satisfy buyers’ diverse tastes, with the marginal cost of redesign increasing in the distance between the basic product and a buyer’s ideal taste. Suitable for modeling customization in certain job shops, delivered pricing loses its validity in the new context of flexible manufacturing, where the notion of a “basic product” becomes ill-defined and all the planned varieties can be produced equally efficiently. This observation motivates the way customization is characterized in our paper.

Flexibility in manufacturing means being able to reconfigure manufacturing resources so as to produce different products efficiently (Sethi and Sethi 1990). Production flexibility is the ability to quickly and economically vary the part assortment for any product that an FMS can produce. An obvious measure for production flexibility is the volume of the universe of products the system is capable of producing (Chatterjee et al. 1984). Here, volume can be expressed by the number of product models or the range of product specifications (Browne et al. 1984, Gerwin 1987). The concept of customization scope, defined later in our model, closely resembles the degree of production flexibility.

Strategic entry deterrence has often been studied. Scale economies and learning curve effects are well-known entry barriers (Bain 1956). Spence (1977) shows that incumbent firms in an industry may carry excess capacity to deter entry. In a spatial framework, Schmalensee (1978) argues that brand proliferation and advertising can serve as credible entry-deterring threats in the breakfast cereal industry. His model relies on very stringent assumptions, including increasing returns at the brand level, brand immobility, and fixed prices, and thus leaves the incumbent with few options other than to “over-proliferate” brands. In contrast, our model assumes constant returns to production and allows firms to vary price. Even with these relaxed assumptions, we find that the incumbent can still deter entry through “over-customization.”

We develop a model of product customization and price discrimination in §2. Sections 3 and 4 examine the customization choices of a monopoly with one
2. Model
We adopt the Salop (1979) model, which allows us to ignore firms’ location decisions. Symmetric location of firms is an equilibrium in the circular model.

2.1. The Product Space and Consumer Preferences
Consumer preferences are uniformly distributed along a circle of unit length. Each consumer is identified by a point that represents her ideal product. Each consumer has a unitary demand subject to reservation price \( r \). A product at arc distance \( y \) away generates a utility of \( r - ty \), where \( t \) is called the fit cost and measures consumers’ sensitivity to product differences. All available products and their prices are public knowledge. A consumer will buy the product that gives her the maximum net surplus. These are common assumptions in the literature on spatial models.

2.2. The Production and Information-Collection Technologies
A firm may choose to produce a standard product or a range of customized products. When the firm produces a single brand, it is still represented as a point on the circle. We assume a production technology with constant returns and normalize marginal cost to zero. When the firm adopts customization and produces a range of products, they are represented as an arc of the circle. The motivation behind such an extension is that, in electronic markets, sellers observe buyers’ preferences by delegating product design to customers (von Hippel 1998, Wind and Rangaswamy 1999) and can tailor products for all consumers whose ideal tastes lie within the capability of its flexible production facility.

When adopting customization, the firm must bear additional fixed costs for acquiring manufacturing flexibility and gathering and processing information. These fixed costs depend on the firm’s scope of customization activities. Flexibility and cost efficiency have in the past been considered as conflicting objectives. The introduction of flexibility into a manufacturing system requires high initial investments with decreasing returns (Gupta and Goyal 1989, Bobrowski and Mabert 1988). When determining the desired degree of flexibility, one should trade off the system’s capability to manufacture a variety of products with the available capital investment. This is why many build-to-order merchants, such as Dell Computers, allow only a limited menu of custom product configurations.

Because we assume consumers are uniformly distributed, the amount of consumer preference data that must be gathered and processed increases proportionately with the scope of customization. Once an information system is set in place, collecting information (whose major activities include consumer tracking and data storage) has relatively steady marginal costs within the capacity of the system, as supported by casual empiricism. For example, Amazon.com and Yahoo! can gather the purchasing and preference profile information of each member of their huge customer pool at roughly the same marginal cost (Business Week 1999). Likewise, Dell takes each additional custom order at an almost identical cost. However, processing the collected information for customization purposes, like data mining and collaborative filtering, often reflects increasing marginal costs, as the number of comparisons required to draw one inference increases with the size of the data set. In addition, even though most learning mechanisms other than custom orders are not perfect, we assume the obtained consumer preference is accurate enough for customization after extensive and expensive data gathering and processing. Combining the costs of information manipulation and production flexibility, we assume that the fixed cost of producing tailored...
Figure 1  A Duopoly of a Customizer and a Conventional Seller

A Customizer’s Direct Marketing Segment (Customization Scope)

A Conventional Seller

Note. L and R are the two end points of the customization scope and also the customizer’s standard products. Buyers in the left and right conventional segments will choose L and R, respectively.

goods in a scope $x$ of the product space is $ax^2 + bx$. Here, $x$ is called the customization scope and measures the firm’s degree of production flexibility. For ease of exposition, we label the quadratic term ($ax^2$) the flexibility cost and label $bx$ the information cost.

We call consumers in the customization scope the direct-marketing segment and call the other adjacent located consumers the conventional segments. Figure 1 shows a duopoly of a customizer and a conventional seller. In a duopoly, both sellers have access to identical customization and information technologies.

2.3. The Pricing Scheme

We next construct a second-degree discriminatory pricing scheme—a scheme in which the firm sets prices for its products, and among all the product-price pairs each consumer picks one that maximizes her surplus.

Due to symmetry of the market, a firm charges a single price $p$ for its standard products L and R (see Figure 2). Because the standard products are used to serve the two conventional segments, $p$ is also called the conventional market price. In the direct-marketing segment, the firm provides personalized products at higher prices. For a buyer in the direct segment, the firm charges the sum of the conventional market price and her fit cost in consuming the closest standard product. In other words, a customized product at distance $y$ ($y < x/2$) away from the closest standard product is priced at $p + ty$.

Such a linear discriminatory pricing scheme is optimal. To see this, denote the price for buyer $y$ as $p(y)$. If $p(y) > p + ty$, buyer $y$ would choose to purchase the closest standard product at price $p$, and
the customization scheme simply collapses. If \( p(y) < p + ty \), then the seller is not extracting the maximum rent. Under the pricing scheme \( p(y) = p + ty \), buyers in conventional segments \([A, L]\) and \([R, B]\) will choose standard products \(L\) and \(R\), respectively, and buyers in the direct-marketing segment will choose the products tailored specifically for them. The linear pricing scheme \( p(y) = p + ty \) (\( y < x/2 \)) therefore, is indeed, optimal and prevents arbitrage among buyers. An observant reader may have noticed that the optimality of the linear pricing scheme is directly driven by the linear fit cost assumption.

Of course, the seller also has two extreme product strategies to consider. The seller can adopt a pure standardization strategy of producing a single brand or a pure customization strategy of selling only customized products at a price of reservation utility \( r \). When the market is not covered, however, producing both custom and standard goods always dominate pure customization. Imagine that the seller adopting pure customization lowers the price of a standard product \((L\) or \(R)\) by a small amount \( \delta \). The seller gains a profit of \((r - \delta)/t\) from the conventional market and loses a profit of \( \delta^2/(2t) \) from the direct-marketing segment. The gain outweighs the loss. Producing both custom and standard goods also outperforms pure standardization under very general conditions, as we shall see shortly.

### 3. A Monopoly with a Single Customization Scope

Salop (1979) and Tirole (1988) analyze pure standardization by a conventional monopoly: when \( t > r \) so that the market is not covered, the monopolist prices at \( r/2 \), earning a profit of \( r^2/(2t) \) and covering a market of \( r/t \). We also assume \( t > r \) throughout this paper for benchmarking purpose. In our model, the monopolist’s optimal product strategy turns out to depend on the cost coefficients \((a\) and \(b)\) representing flexible manufacturing and information technologies. Under certain conditions (see Lemma 1, Case 1 below), the monopolist may still find pure standardization optimal.

As customization technologies improve and their cost coefficients decrease, the monopolist begins to offer customized products. When the customization cost coefficients are not low enough (as stated in Case 2 of Lemma 1), the monopolist cannot charge any consumer her reservation price even though the product may be exactly what the consumer desires. We next analyze the case in which the market is not covered and the highest price of customized products is below \( r \).

When adopting customization, the seller has two sources of profits: the two conventional segments and the direct-marketing segment. If the conventional market price is \( p \), the length of each conventional segment is \((r - p)/t \). The seller’s profits from the conventional segments are \( 2p(r - p)/t \). The profits from the direct-marketing segment are

\[
2 \int_0^{x/2} (p + ty) dy - ax^2 - bx,
\]

where \( x \) is the customization scope. The monopoly’s problem is to choose \( x \) and \( p \) to maximize its total profits. For expositional clarity, we solve its problem in two steps.

For a fixed customization scope \( x \), the optimal price \( p \) maximizes its revenue:

\[
\text{Max}_{p} 2p \frac{r - p}{t} + 2 \int_0^{x/2} (p + ty) dy. \tag{3.1}
\]

The first-order condition yields \( p = (2r + tx)/4 \). Plugging this price back into (3.1) and incorporating the customization costs, we get the monopoly profits as a function of \( x \):

\[
\pi_{m1}(x) = \left(\frac{3t}{8} - a\right)x^2 + \left(\frac{r}{2} - b\right)x + \frac{r^2}{2t}. \tag{3.2}
\]

The second-order condition of (3.2) requires \( a \geq 3t/8 \). Maximizing (3.2) gives the optimal customization scope

\[
x^*_{m1} = \frac{2(r - 2b)}{8a - 3t}
\]

and conventional market price

\[
\frac{r + t(r - 2b)}{2} + \frac{2(r - 2b)}{2(8a - 3t)}. \tag{3.3}
\]

The monopolist earns profits of

\[
\frac{r^2}{2t} + \frac{(r - 2b)^2}{2(8a - 3t)}.
\]
and covers a market of
\[ \frac{r}{t} + \frac{r - 2b}{8a - 3t}. \]
Here, we need \( a \geq 3t(r - b)/(4r) \) for the highest price of custom products to be less than \( r \) and \( b \leq r/2 \) to have a nonnegligible customization scope. These two conditions jointly imply \( a \geq 3t/8 \). When \( t > 4r/3 \), the circular market is not covered. These results are stated in Lemma 1, Case 2.

Two additional cases arise as the customization cost coefficients further decrease, but their analysis is not detailed here due to space limitation. The monopolist’s optimal product and pricing strategies in all scenarios are presented in Lemma 1 and depicted in Figure 3.

**Lemma 1.**

**Case 1.** When
\[ a \geq \frac{3t}{2} \left(1 - \frac{b}{r}\right)^2 \quad \text{and} \quad b > \frac{r}{2}, \]
the monopolist only offers a single standard product.

**Case 2.** When
\[ a \geq \frac{3t}{4r}(r - b), \quad b < \frac{r}{2}, \quad \text{and} \quad t \geq \frac{4}{3}r, \]
the monopolist offers both standard and customized goods, and the highest price of custom products is below \( r \).

**Case 3.** When
\[ a \leq \min \left\{ \frac{3t}{2} \left(1 - \frac{b}{r}\right)^2, \frac{3t(r - b)}{2(3t - 2r)} \right\}, \quad a \geq \frac{3t(r - b)}{2(3t - 2r)}, \]
and
\[ t \geq \frac{4}{3}r, \]
the monopolist offers both standard and customized goods and the highest price of custom products reaches \( r \). The optimal customization scope is \((r - b)/2a\), and the conventional market price is \( 2r/3 \).

**Case 4.** When
\[ a \leq \min \left\{ \frac{3t}{2} \left(1 - \frac{b}{r}\right)^2, \frac{3t(r - b)}{2(3t - 2r)} \right\}, \]
the monopolist offers customized goods at price \( r \) for all customers in the circular market (the market is covered).

We continue to analyze the welfare impacts of customization in Lemma 1, Case 2. For a given customization scope \( x \) and conventional market price \( p \), consumer surplus is
\[
CS(x, p) = \frac{(r - p)^2}{t} + (r - p)x - \frac{t}{4}x^2. \tag{3.3}
\]
Substituting the monopoly price \( p = (2r + tx)/4 \) into (3.3), we have consumer surplus as a function of
customization scope $x$:  

$$CS(x) = \frac{1}{16} \left( \frac{4}{t} r^2 + 4rx - 7tx^2 \right).$$  

(3.4)

Maximizing (3.4) gives the customization scope that maximizes consumer surplus under monopoly pricing $x^*_c = 2r/7t$. In other words, if the consumers could choose the scope of customization, knowing that prices are set by a monopolist, then they as a group would pick $x^*_c$. We use this benchmark to define over- and underinvestment in customization from the consumers’ perspective.

Definition 1. The monopoly underinvests (overinvests) in customization if $x^*_m < x^*_c$ ($x^*_m > x^*_c$).

Proposition 1. Assume $a \geq (3t/4r)(r - b)$, $b < r/2$, and $t \geq (4/3)r$. Then,

(1) Monopoly response to technological advances. When customization and information technologies advance, i.e., when the customization cost coefficients $a$ and $b$ decrease, the monopolist raises the price of the standard goods and expands its customization scope.

(2) Consumer surplus. Consumer surplus is higher than no customization when $a \geq (t/16r)(13r - 14b)$ and lower otherwise. When $a \geq (t/4r)(5r - 7b)$, the monopolist underinvests in customization and consumer surplus improves with advances in customization technologies. When $(3t/4r)(r - b) \leq a \leq (t/4r)(5r - 7b)$, the monopolist overinvests in customization and consumer surplus deteriorates with advances in customization and information technologies.

(3) Social surplus. Social surplus is higher than no customization when $a \geq 3t/4$ and lower otherwise. When the underlying technologies advance, social surplus increases when $a \geq 9t/8$ and decreases otherwise.

Raising the conventional market price lowers the sales of the standard products but also permits higher prices for the custom products. In fact, with more advanced technologies, the seller will deliberately abandon some buyers in the conventional market to better exploit an expanded direct-marketing segment. Customization not only benefits the monopolist itself but may also improve consumer surplus and total social surplus. Contrary to common belief, consumers as a group may not be worse off with new developments in customization and information technologies in a monopoly setting, because in maximizing its profits the monopoly may either under- or overinvest in customization from the perspective of maximizing consumer surplus, depending on the technological status. Technological improvement reduces total fit cost and makes a larger market covered, so it may drive up consumer surplus.

Mass customization will not be a feasible product strategy if the supporting technologies are not reasonably economical. In our model, for the seller to adopt customization, the coefficient of the information-gathering cost, $b$, must be sufficiently low ($b < r/2$ in this case). This is where the Internet technologies have had a great impact by providing useful learning mechanisms. Recording purchases and visits in logs and databases, data mining, collaborative filtering, and custom orders are all instances of information technologies for learning purposes. Furthermore, breakthroughs in flexible manufacturing allow the use of this information for mass customization of consumer goods in many categories, such as apparel and furniture.

4. A Monopoly with Two Customization Scopes—A Benchmark

We now examine a market-covering monopoly with two symmetric customization scopes as a benchmark for a duopoly. Such a two-scope monopoly represents a firm offering two distinct lines of customized products requiring separate flexible installations. Firms in many industries commonly offer multiple product lines. For instance, Ford Motor Company produces several lines of vehicles, each of which may be customized (Warner and Bank 2000). Because the market is always covered in a duopoly, any sensible benchmark must consider the market-covered case. Denote $x$ and $p$ as the length of each customization scope and the conventional market price, respectively. The profits of the two-scope monopoly are

$$p + \frac{1}{2}tx^2 - 2(ax^2 + bx).$$  

(4.1)
Notice that, for any \( x \), the monopoly does the best by choosing the highest price \( p \) that still keeps the circular market covered. That is,
\[
2 \left( x + 2 \frac{-p}{t} \right) = 1. \quad (4.2)
\]
The monopoly’s optimal price is \( p = r - (t/4) \) at \( 2x \). Simplifying (4.1) with this expression, we obtain the monopoly’s profits as a function of \( x \) only,
\[
\pi_{m2}(x) = -2 \left( a - \frac{t}{4} \right) x^2 - 2 \left( b - \frac{t}{4} \right) x + r - \frac{t}{4}. \quad (4.3)
\]
The second-order condition requires \( a \geq t/4 \). Maximizing (4.3) leads to Lemma 2.

**Lemma 2.** The monopoly sets the length of each customization scope at
\[
x_{m2} = \frac{t - 4b}{2(4a - t)}
\]
and chooses conventional market price
\[
p_{m2} = r - \frac{(2(a + b) - t)t}{2(4a - t)}.
\]

5. A Duopoly with Simultaneous Customization Investments

Before analyzing the competitive implications of customization, we briefly revisit the degenerate case of a conventional duopoly, in which each firm offers a single brand (Salop 1979): the two firms set their prices at \( t/2 \), each making a profit of \( t/4 \). Consumer surplus is \( r - 5t/8 \), and social surplus is \( r - t/8 \).

Consider a two-stage game in which firm 1 and firm 2 choose customization scopes \( x_1 \) and \( x_2 \) simultaneously in stage 1 and choose conventional market prices \( p_1 \) and \( p_2 \) simultaneously in stage 2. Assume symmetric location of the customization scopes.

**Lemma 3.** In any sub-game perfect equilibrium (SPE) of the two-stage customization-pricing game, the two firms’ customization scopes are not contiguous.

Lemma 3 lets us focus on the case in which the two firms’ customization scopes do not overlap. Correspondingly, certain constraints are imposed on the parameters (see the restated lemmas and propositions in the Appendix) to have a more tractable analysis. This game was solved in both Dewan et al. (2000) and Jing (2001), and its solution is stated in Lemma 4.

**Lemma 4.** The two-stage game in customization scopes and conventional market prices has a unique SPE
\[
(x_1, x_2, p_1, p_2) = \left( \frac{t - 6b}{12a - 3t'}, \frac{t - 6b}{12a - 3t'}, \frac{t}{2}, \frac{t}{2} \right)
\]
At equilibrium, the two sellers earn identical profits
\[
\pi_1 = \pi_2 = \frac{18(b^2 + at) - 5t^2}{18(4a - t)}.
\]

**Proposition 2.** Simultaneous adoption of customization results in sellers producing goods that are closer in attribute space, i.e., it reduces the differentiation between the sellers’ standard products. It does not aggravate price competition: The price for the standard goods remains at \( t/2 \), the price in absence of customization. When \( b < t/6 \) (\( b > t/6 \)), the seller’s profit decreases (increases) as a decreases.

It is counterintuitive that customization does not reduce price competition in the conventional market. In a conventional duopoly, the brands are \( 1/2 \) apart and the price is \( t/2 \). When the sellers begin to offer customized products, their standard products at the edges of their customization scopes get closer in the attribute space. Existing theory on product differentiation would predict that price competition intensifies as the differentiation between the standard products diminishes (the familiar Bertrand competition is an extreme case when the products are identical). Yet this does not happen in our model, as the price remains at \( t/2 \). When competing in the conventional market, each firm takes into account the fact that the prices of customized products explicitly depend on the conventional market price. In this sense, price discrimination on customized products relaxes the price competition between standard products.

With more advanced customization technologies available, the firms will expand their customization scopes. Each seller can collect higher revenue from his customized products, but the customization cost also increases. The revenue gain from customized products cannot exceed the escalation in technological expenditure. Consequently, firms in a competitive setting do not always benefit from customization.

Next, we compare this equilibrium with the choices of the two-facility monopolist discussed in §4. As one would expect, the price and profits of each
PRODUCT CUSTOMIZATION AND PRICE COMPETITION ON THE INTERNET

duopolist are lower than those of the monopolist (on a per facility basis). Similarly, one would also expect the duopolist to compete for customers by over-customizing, but this is not always the case.

**Proposition 3.** When \( a \geq (1/4)(3t-8b) \) and \( b < t/6 \), each duopolist underinvests in customization compared with the choice of the two-facility monopolist.

Under the conditions of Proposition 3, we can readily verify that the conventional market price in the duopoly, \( t/2 \), is lower than that of the two-facility monopoly,

\[
r = \frac{(2a+b)-t}{2(4a-t)} t.
\]

The price premium on customized products is also hamstrung by price competition in the conventional market. Competition reduces firms’ incentive to invest by restricting the returns from customization. Proposition 4 shows the welfare impacts of customization in a duopoly.

**Proposition 4.**
1. With customization, consumer surplus is higher than in the conventional duopoly. Consumer surplus first increases and then decreases when flexible manufacturing and information gathering technologies improve.

2. When

\[
b < \frac{t}{6} \quad \text{and} \quad a \geq \frac{(2t-9b)t}{4(t-3b)},
\]

social surplus is higher than in the conventional duopoly. When

\[
b < \frac{t}{6} \quad \text{and} \quad \frac{1}{12}(5t-12b) \leq a < \frac{(2t-9b)t}{4(t-3b)},
\]

social surplus is lower than in the conventional duopoly.

After both firms adopt customization, consumers are better off as a group, but consumer surplus is not monotonically increasing in the cost parameters. When technologies advance, the prices of custom products rise but the expansion of customization scopes also reduces the fit cost of buyers in the conventional segments. The adjustment in consumer surplus thus depends on the relative magnitude of these two effects.

It is interesting to note that

\[
x_1 = x_2 = \frac{t-6b}{12a-3t} \rightarrow 0 \quad \text{and}
\]

\[
\pi_1 = \pi_2 = \frac{18b^2 + 4at - 5t^2}{18(4a-t)} \rightarrow \frac{t}{4}
\]

as \( a \rightarrow \infty \). When high cost of customization precludes the use of these technologies, our results are the same as Salop. Further comparison with results in Salop (1979) reveals that both firms have lower profits. Here we see a case of technology that, while increasing the overall surplus, causes the profits of the technology deployers to decrease. This supports the “mismeasurement” hypothesis by Brynjolfsson (1993) for the “IT productivity paradox.”

6. A Duopoly with Sequential Customization Investments

Firms differ in their capabilities of organizational learning and their readiness to adopt new technologies. Even though firms selling only a standard product cannot obtain a first-mover advantage, customization can create an advantage for an early adopter, as we show next. Consider a three-stage game of sequential adoption of customization. Firm 1 picks customization scope \( x_{1\text{seq}} \) in stage 1, and firm 2 picks customization scope \( x_{2\text{seq}} \) in stage 2 after observing seller 1’s choice. Finally, both firms choose their conventional market prices \( p_1 \) and \( p_2 \) simultaneously in stage 3. We solve this game backwards, starting with the last stage.

6.1. Stage 3: The Conventional Market Pricing Game

In stage 3, customization costs are sunk and the firms observe their customization scope choices \( x_{1\text{seq}} \) and \( x_{2\text{seq}} \). Firm \( i \) chooses price \( p_i \) to maximize its revenue

\[
x_{i\text{seq}} \left( p_i + \frac{t}{4} x_{i\text{seq}} \right) + p_i \left( \frac{1}{2} + \frac{p_j - p_i}{t} - \frac{x_{i\text{seq}} + x_{j\text{seq}}}{2} \right),
\]

where the first term is the revenue from customized goods and the second term is the revenue from the standard goods.

The equilibrium price can be easily found:

\[
p_i = \frac{1}{6} t (3 + x_{i\text{seq}} - x_{j\text{seq}}).
\]
Hence, the firm with a larger customization scope will set a higher conventional market price.

6.2. Stage 2: Firm 2 Chooses Its Customization Scope

In stage 2, firm 1’s choice $x_{1\text{seq}}$ is known, and firm 2 anticipates the third-stage outcome and picks customization scope $x_{2\text{seq}}$ to maximize its profits. Substituting (6.2) into (6.1), we obtain firm 2’s stage-2 objective

$$
\pi_{2\text{seq}}(x_{2\text{seq}}) = \frac{1}{36} t \left[ 10x_{2\text{seq}}^2 - 2x_{2\text{seq}}(x_{1\text{seq}} - 3) + (x_{1\text{seq}} - 3)^2 \right] - ax_{2\text{seq}}^2 - bx_{2\text{seq}}.
$$

The second-order condition of (6.3) requires $a > 5t/18$. Solving (6.3) gives firm 2’s optimal customization scope response

$$
x_{2\text{seq}}(x_{1\text{seq}}) = \frac{18b + (x_{1\text{seq}} - 3)t}{-36a + 10t}.
$$

6.3. Stage 1: Firm 1 Chooses Its Customization Scope

Anticipating the outcomes in the ensuing stages, firm 1 picks customization scope $x_{1\text{seq}}$ to maximize its profits in stage 1

$$
\pi_{1\text{seq}}(x_{1\text{seq}})
= \frac{1}{36} t \left[ \frac{(18b + t(-3 + x_{1\text{seq}}))}{-36a + 10t} - 3 \right]^2 
+ \left( \frac{108a + 18b + t(-33 + x_{1\text{seq}}))x_{1\text{seq}} + 10x_{1\text{seq}}^2}{18a - 5t} \right]
- ax_{1\text{seq}}^2 - bx_{1\text{seq}}.
$$

The second-order condition requires that $a \geq \frac{21 + \sqrt{5}}{72} t$. Solving (6.5) gives firm 1’s optimal customization scope

$$
x_{1\text{seq}}^* = \frac{2b}{-4a + t} + \frac{3(36a - 11t)t}{1296a^2 - 756at + 109t^2}.
$$

Substituting $x_{1\text{seq}}^*$ into (6.4), we obtain firm 2’s optimal customization scope

$$
x_{2\text{seq}}^* = \frac{2b}{-4a + t} + \frac{36(3a - t)t}{1296a^2 - 756at + 109t^2}.
$$

The two sellers’ stage-3 prices are

$$
p_1^* = \frac{(36a - 11t)(18a - 5t)t}{1296a^2 - 756at + 109t^2}
$$

and

$$
p_2^* = \frac{54(4a - t)(3a - t)t}{1296a^2 - 756at + 109t^2}.
$$

Under the condition $a \geq (4/9)t$ (see restated Lemma 5 in the Appendix), the second-order conditions of both (6.3) and (6.5) are satisfied

$$
\left( \frac{4}{9} t > \frac{21 + \sqrt{5}}{72} t > \frac{5}{18} t \right).
$$

We summarize the solution to this three-stage game in Lemma 5.

**Lemma 5.** The three-stage game of sequential customization adoption has a unique SPE as described in (6.6)–(6.9).

At the sequential equilibrium, the two firms’ profits are

$$
\pi_{1\text{seq}} = \frac{b^2}{4a - t} + \frac{(36a - 11t)t}{5184a^2 - 3024at + 436t^2}
$$

and

$$
\pi_{2\text{seq}} = \frac{b^2}{4a - t} + \frac{648(18a - 5t)(4a - t)(-3a + t)^2t}{(1296a^2 - 756at + 109t^2)^2}.
$$

The difference between their profits is

$$
\pi_{1\text{seq}}^* - \pi_{2\text{seq}}^* = \frac{t^2(2592a^2 - 1548at + 229t^2)}{4(1296a^2 - 756at + 109t^2)^2},
$$

which is greater than zero when $a \geq (4/9)t$. The first adopter makes a higher profit than the second adopter. We can also verify that the first adopter covers a larger market.

**Proposition 5.** The first adopter chooses a larger customization scope and charges a higher conventional market price than the second adopter. When customization and information technologies advance (i.e., customization cost coefficients $a$ and $b$ decrease), the first and second adopters will raise and lower their conventional market prices, respectively, and the difference between their customization scopes will increase.
The first mover secures its advantage by investing more in customization, and it can charge higher prices and grab a larger market share. The leader can afford to abandon more sales of standard goods because it obtains higher mark-ups from the customized products by charging a higher conventional market price. Such a first-mover advantage is due to the leader's freedom to choose a more favorable customization scope to manipulate the follower's customization scope and price (see (6.2) and (6.4)). Besides, more advanced underlying technologies imply a greater first-mover advantage.

7. Using Customization to Deter Entry

The analysis so far assumes that sellers have already entered the market and ignores costs of entry. In this section, we assume that each firm also faces an additional cost \( c \) of entering the market. Two additional scenarios other than sequential duopoly are now possible, and the first mover (firm 1) may do even better. In the first, entry of firm 2 is blocked, as firm 1’s monopoly customization scope automatically eliminates all entry possibilities. In this case, we call firm 1 a natural monopoly. In the second scenario, entry is deterred when firm 1 deliberately chooses a customization scope larger than the monopoly choice to preempt firm 2’s entry. In this case, we call firm 1 a strategic monopoly. In the final scenario, firm 1’s best choice is to accommodate entry, and the resulting market structure is a sequential duopoly of the kind analyzed in §6.

To find firm 1’s best customization scope choice, we need to identify the monopoly profit as a function of its customization scope \( x \). Incorporating entry costs into (3.2), we have

\[
\pi_{m1}(x) = \left( \frac{3t}{8} - a \right) x^2 + \left( \frac{r}{2} - b \right) x + \frac{r^2}{2t} - c. \tag{7.1}
\]

Firm 1’s monopoly customization scope is

\[
x_{m1} = \frac{2(r - 2b)}{8a - 3t}.
\]

We next determine firm 1’s entry-deterring customization scope choice \( x_{1d} \), the smallest customization scope that makes entry unattractive to firm 2. Firm 2 takes firm 1’s scope, \( x_{1eq} \), into account when picking its customization scope, as shown in Equation (6.4). Substituting (6.4) into (6.3) and considering the costs of entry, we obtain firm 2’s optimal profit as a function of \( x \):

\[
\pi_{2eq}(x) = \frac{1}{8(18a - 5t)} \left[ t(4a - t)(x - 3)^2 + 4bt(x - 3) + 36b^2 \right] - c. \tag{7.2}
\]

The smaller root to \( \pi_{2eq}(x) = 0 \) is the entry-deterring customization scope

\[
x_{1d} = \frac{12a - 2b - 3t}{4a - t} - 2\sqrt{\frac{1}{2}\sqrt{(18a - 5t)}(4a - t)c - b^2}}{(4a - t)t}. \tag{7.3}
\]

As entry cost \( c \) increases, firm 1’s entry-detering customization scope will decrease. The higher the entry barrier, the easier deterrence becomes. When \( x_{m1} \geq x_{1d} \), firm 1 can act as a natural monopoly by choosing \( x_{m1} \). Equivalently, entry is blockaded when \( r \geq r_b \) if we adopt the notation

\[
r_b = \frac{8a - 3t}{2} x_{1d} + 2b. \tag{7.4}
\]

But what if \( x_{m1} < x_{1d} \)? Recall from §6 that \( \pi_{1eq}^* \) is firm 1’s optimal profit in the sequential equilibrium, excluding entry cost \( c \). When \( \pi_{m1}(x_{1d}) > \pi_{1eq}^* - c \), firm 1’s best choice is to pick \( x_{1d} \) to deter firm 2’s entry and act as a strategic monopoly; otherwise, firm 1 should accommodate firm 2’s entry. Solving the equation \( \pi_{m1}(x_{1d}) = \pi_{1eq}^* - c \), we obtain the critical value of reservation utility:

\[
r_d = \frac{1}{2(4a - t)^2} \left( \frac{f_1 + 2\sqrt{2}\sqrt{f_2(f_3 + f_4)}}{1296a^2 - 756at + 109t^2} \right), \tag{7.5}
\]

where

\[
f_1 = \sqrt{t}(-4a + t),
\]

\[
f_2 = (12a - 2b - 3t)\sqrt{t} - 2\sqrt{2(18a - 5t)}(4a - t) - b^2, \]

\[
f_3 = -3(4a - t)\sqrt{2(18a - 5t)}[c(4a - t) - b^2] - (1296a^2 - 756at + 109t^2), \quad \text{and}
\]

\[
f_4 = (4a - t)^3(1296a^2 - 756at + 109t^2). \]
Using excessive customization to deter entry is a variant of the brand proliferation strategy discussed in Schmalensee (1978), where the incumbent firms can “crowd out” potential entrants by properly spacing new brands in the product attribute space. A first mover in electronic markets can position itself as a versatile vendor of a sufficiently large span of product varieties, leaving no profitable niche for the new entrant.

Using customization to deter entry has a different rationale than economies of scale (Bain 1956), whose effectiveness is derived from the incumbent’s unit-cost advantage when producing a significant fraction of total industry demand. In contrast, the effectiveness of using customization for entry deterrence critically hinges on the fixed entry cost \( c \), the setup cost besides investments in customization. Absent such an entry cost, firm 2 can always profitably enter as long as the incumbent’s customization scope does not cover the whole product space.

8. Conclusions and Future Extensions

The Internet and other information-processing technologies allow sellers to better understand each customer’s needs and wants, facilitating market provision of customized consumer goods. Devising a spatial model of customization, we evaluate firms’ product customization strategies in an electronic market and obtain interesting insights on their product mix, pricing, and consumer surplus, as well as technologies’ impacts on these strategic variables. The novelty of our model derives from a distinct cost-structure assumption that customization requires a fixed initial investment with decreasing returns but can produce each planned variety with equal efficiency. This cost structure fits most modern flexible manufacturing settings well. The customized product is priced high enough to make the customer indifferent between purchasing the customized product and the standard product; i.e., customization allows the seller to extract more surplus from the consumer. Consequently, a monopoly will raise the prices of both types of products to take better advantage of advances in customization technologies.

In a duopoly, customization reduces the differentiation between the sellers’ standard offerings. The sellers are worse off and consumers benefit from customization, but consumer surplus does not monotonically increase. Moreover, the timing of adopting customization is critical. An early adopter can always secure and sustain an advantage and may be able to keep out potential competitors by investing properly in customization.

Our model is a highly simplified, abstract characterization of customization and has several limitations.
First, the circular product space and the cost structure seem reasonable for most products suitable for flexible manufacturing, such as clothes, furniture, cars, and computers, but may not apply for certain information goods and services such as customized newspapers and travel packages. Second, our model is a single-period one and does not capture intertemporal learning by the first mover about customers’ tastes. In a multiperiod setting, learning by the sellers may foster switching costs for customers and form another source of first-mover advantage for repeatedly purchased items such as clothes. One meaningful direction for future research is, thus, to build a multiperiod model and examine the impacts of learning about customer tastes on competition. We hypothesize that this extension will reveal more dynamics about the first mover’s advantage. Finally, we assume that product differentiation is conducted in an attribute along a single horizontal dimension. In reality, customizing firms also distinguish themselves along other dimensions, such as brand name, quality, and delivery time. The predictions of our model therefore may not be duplicated in all real-world settings.

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Appendix

Proof of Proposition 1. (1) This part is straightforward to verify.

(2) First notice that when

\[ a \geq \frac{t}{4r}(5r - 7b) \quad \left( a < \frac{t}{4r}(5r - 7b) \right), \]

we have \( x_{\text{a}} \leq x_{\nu} \) (\( x_{\text{a}} > x_{\nu} \)). The consumer surplus is

\[ u = (r - p') \frac{r - p'}{t} + 2 \int_{\bar{y}}^{\hat{y}} (r - p' - y) dy \]

\[ = \frac{1}{(8a - 3t)^2} \left[ 16a^2 - 8ar(b + r) + \left( (b - 10r + r^2) \right)^2 \right]. \]

We can check that this consumer surplus is greater than that without customization, \( r^2/4t \), when \( a \geq (t/16r)(13r - 14b) \) and lower otherwise. We also have

\[ \frac{\partial u}{\partial a} = \frac{8(2b - r)(4ar + 7bt - 5rt)}{(8a - 3t)^3} \]

and

\[ \frac{\partial u}{\partial b} = -\frac{2(4ar + 7bt - 5rt)}{(8a - 3t)^2}. \]

When

\[ a \geq \frac{t}{4r}(5r - 7b) \quad \text{and} \quad b < \frac{r}{2}, \]

we have

\[ \frac{\partial u}{\partial a} < 0 \quad \text{and} \quad \frac{\partial u}{\partial b} < 0. \]

When

\[ \frac{3t}{4r}(r - b) \leq a \leq \frac{t}{4r}(5r - 7b) \quad \text{and} \quad b < \frac{r}{2}, \]

we have

\[ \frac{\partial u}{\partial a} > 0 \quad \text{and} \quad \frac{\partial u}{\partial b} > 0. \]

The response of consumer surplus to developments in technologies then follows.

(3) The social surplus is

\[ w = r \left( x^2 \frac{2(r - p')}{t} - ax^2 - bx = \frac{r^2}{t} + (r - 2b)^2(4a - 3t) \right)^2, \]

which is greater than that without customization, \( r^2/4t \), when \( a \geq 3t/4 \) and lower otherwise. The response of social surplus to flexibility cost follows from

\[ \frac{\partial w}{\partial a} = \frac{-4(r - 2b)^2(8a - 9t)}{(8a - 3t)^3}. \]

Lemma 2 (Revised). When

\[ (a, b) \in \left\{ (a, b) \mid a \geq \frac{1}{4}(3t - 8b), \ b < \frac{t}{4} \right\} \]

\[ \cup \left\{ (a, b) \mid a < \frac{1}{4}(3t - 8b), \ b > \frac{t}{4} \right\} \quad \text{and} \]

\[ r > \frac{t}{4(4a - t)}(8a + 4b - 3t), \]

the two-scoped monopoly picks customization scope length

\[ x_{\text{a}} = \frac{t - 4b}{2(4a - t)} \]

and chooses conventional market price

\[ p_{\text{a}} = r - \frac{(2a + b - 1)t}{2(4a - t)}. \]

Proof of Lemma 2. When

\[ (a, b) \in \left\{ (a, b) \mid a \geq \frac{1}{4}(3t - 8b), \ b < \frac{t}{4} \right\} \]

\[ \cup \left\{ (a, b) \mid a < \frac{1}{4}(3t - 8b), \ b > \frac{t}{4} \right\}, \]

the peak price for customized products is less than \( r \), and the customization scope is strictly less than \( t/2 \). When

\[ r > \frac{t}{4(4a - t)}(8a + 4b - 3t), \]

the circular market is covered. □
Proof of Lemma 3. We prove this lemma in two steps by contradiction. First, we show that overlapping customization scopes are not a Nash equilibrium. Suppose there is an equilibrium in which the two sellers’ customization scopes overlap. Bertrand competition implies that the prices for the customized products in the overlapped segment are zero. Either firm (firm 1 say) can be strictly better off by retreating to the point where the two firms’ customization scopes just touch, maintaining his revenue but reducing customization costs.

Next, we show that the state where the two customization scopes are contiguous is not Nash. Suppose firm 1 (2) chooses customization scope \( x_1 (x_2) \) and \( x_1 + x_2 = 1 \). Denote \( E \) and \( F \) to be the touching points of the two firms’ customization scopes. Then, the prices of products \( E \) and \( F \) are zero. Now imagine firm 1 retreats slightly by \( \xi \) on both ends and denote his standard products (the new endpoint of his customization scope) as \( G \) and \( H \). The new equilibrium prices of the standard products are found to be

\[
p_c = p_H = \frac{t}{6} (3 + x_2 - 2\xi - x_1) > \frac{t}{3}
\]

and

\[
p_s = p_H = \frac{t}{6} (3 + x_2 - x_1 + 2\xi) > \frac{t}{3}
\]

because \(-1 < x_2 - (x_1 - 2\xi) < 1\). Retreating makes firm 1 strictly better off; he loses \(2\xi\) in sales but can charge a much higher price on each of his remaining product offerings. We have completed the proof of Lemma 3. □

Lemma 4. When

\[
(a, b) \in \left\{ (a, b) \mid a > \frac{1}{12} (5t - 12b), \ b < \frac{t}{6} \right\} \\
\cup \left\{ (a, b) \mid a < \frac{1}{12} (5t + 12b), \ b > \frac{t}{6} \right\}
\]

and

\[
r > \max \left\{ \frac{t}{2} \left( 1 + \frac{t-6b}{12a-3t} \right), \frac{t}{2} \left( \frac{3}{2} - \frac{t-6b}{12a-3t} \right) \right\}
\]

the two-stage customization-pricing game has a unique sub-game perfect equilibrium

\[
\left\{ \frac{t-6b}{12a-3t}, \frac{t-6b}{12a-3t}, \frac{t}{2} \right\}.
\]

Proof of Lemma 4. When

\[
(a, b) \in \left\{ (a, b) \mid a > \frac{1}{12} (5t - 12b), \ b < \frac{t}{6} \right\} \\
\cup \left\{ (a, b) \mid a < \frac{1}{12} (5t - 12b), \ b > \frac{t}{6} \right\}
\]

sellers have nonoverlapping customization scopes. When

\[
r > \frac{t}{2} \left( 1 + \frac{t-6b}{12a-3t} \right),
\]

the highest price of customized products is less than \( r \). When

\[
r > \frac{t}{2} \left( \frac{3}{2} - \frac{t-6b}{12a-3t} \right),
\]

the market is covered, and the two sellers are indeed competing. The rest of the proof is self-evident from the deduction in the text. □

Proof of Proposition 2. When sellers adopt customization, there is less differentiation between their standard products due to the presence of the customized product spectrums. From Lemma 4, the conventional market price is still \( t/2 \), the same as the price in a conventional duopoly. The rest of the proposition follows from

\[
\frac{\partial \pi_a}{\partial a} = \frac{\partial \pi_b}{\partial a} = \frac{(t+6b)(t-6b)}{9(4a-t)^2}.
\]

Proof of Proposition 3. The proof is done by directly comparing the customization scopes in the two-facility monopoly and the simultaneous duopoly. □

Proposition 4 (Restated). (1) With customization, consumer surplus is higher than in the conventional duopoly. Consumer surplus is increasing in information collection cost \( b \) when \( b < (1/24)(7t-12a) \) and decreasing in \( b \) when \( b > (1/24)(7t-12a) \). If \( b < t/6 \), consumer surplus is increasing in \( a \) when

\[
\frac{1}{12} (5t - 12b) - a < \frac{1}{12} (7t - 24b)
\]

and decreasing in \( a \) when \( a > (1/12)(7t - 24b) \). If \( b > t/6 \), consumer surplus is increasing in \( a \) when \( a < (1/12)(7t - 24b) \) and decreasing in \( a \) when

\[
\frac{1}{12} (7t - 24b) - a < \frac{1}{12} (5t - 12b).
\]

(2) Suppose \( b < t/6 \). When

\[
a \geq \frac{(2t-9b)t}{4(t-3b)},
\]

social surplus is higher than in the conventional duopoly. When

\[
\frac{1}{12} (5t - 12b) - a < \frac{(2t-9b)t}{4(t-3b)},
\]

social surplus is lower after customization.

Proof of Proposition 4. (1) Consumer surplus when both sellers adopt customization scope \( x \) is

\[
r - \frac{t}{2} - \frac{t}{8} \left( (2x)^2 + (1-2x)^2 \right) > r - \frac{5t}{8}
\]

for all \( 0 < x < 1/2 \). Consumers thus are always better off than in a conventional duopoly. The value of consumer surplus is

\[
CS_x = \frac{r - \left[ 144(5a^2 + 2ab + b^2) - 24(17a + 7b)t + 65^2 \right]t}{72(4a-t)^2}.
\]

The monotonicity of consumer surplus in \( a \) and \( b \) follows from the respective derivatives:

\[
\frac{\partial CS_x}{\partial b} = \frac{t(-12(a+2b)+7t)}{3(4a-t)^2}, \quad \frac{\partial^2 CS_x}{\partial b^2} = \frac{-8t}{(4a-t)^2} < 0,
\]

\[
\frac{\partial CS_x}{\partial a} = \frac{2(12a+24b-7t)(6b-t)t}{9(4a-t)^3}, \quad \text{and}
\]

\[
\frac{\partial^2 CS_x}{\partial a^2} = \frac{-16(4a+12b-3t)(6b-t)t}{3(4a-t)^3}.
\]
The second-order condition is also satisfied when determining the monotonicity of consumer surplus in $a$.

(2) The social surplus is
\[
SS_a = r - \frac{t}{8} \left( \frac{(t-6b)(9b-2t)+4a(t-3b)}{9(4a-t)^2} \right),
\]
from which the proof follows. □

**Lemma 5 (Restated).** When
\[
a > \frac{4}{9} t, \quad b > \frac{4a-t}{t} \left( \frac{(36a-111)(36a-7t)}{2(1296a^2-756at+1097t^2)} - r \right).
\]
and
\[
b < \min \left\{ \frac{18(3a-t)(4a-t)}{1296a^2-756at+1097t^2} \right\}
\]
the three-stage game of sequential customization scope choices has a unique sub-game perfect equilibrium as described in (6.6)–(6.9).

**Proof of Lemma 5.** When $a > (4/9)t$, sellers have nonoverlapping customization scopes. When
\[
b > \frac{4a-t}{t} \left( \frac{(36a-111)(36a-7t)}{2(1296a^2-756at+1097t^2)} - r \right),
\]
the highest price of customized products is below $r$. When
\[
b < \min \left\{ \frac{18(3a-t)(4a-t)}{1296a^2-756at+1097t^2} \right\},
\]
sellers have positive customization scopes, and they both indeed adopt customization. □

**Proof of Proposition 5.** The difference between the two seller’s customization scopes is
\[
x_{2\infty} - x_{1\infty} = \frac{3t^2}{1296a^2-756at+1097t^2}.
\]
We can easily check that $1296a^2-756at+1097t^2 > 0$ when $a > (4/9)t$. We therefore have $x_{1\infty} - x_{2\infty} > 0$. Seller 1’s conventional market price is higher than that of seller 2 because
\[
p_1 > p_2 = \frac{t^2}{1296a^2-756at+1097t^2} > 0.
\]
The remaining part of Proposition 5 follows from the facts that
\[
\frac{\partial p_1}{\partial a} = -\frac{54(24a-7t)t^3}{(1296a^2-756at+1097t^2)^2} < 0, \quad \frac{\partial p_2}{\partial a} = -\frac{\partial p_1}{\partial a} > 0,
\]
and
\[
\frac{\partial(x_{1\infty} - x_{2\infty})}{\partial a} = -\frac{324(24a-71)t^4}{(1296a^2-756at+1097t^2)^2} < 0
\]
under the conditions of Lemma 5. □

References


